A Comparative Study On Decoupling Methods For Time-Delay Systems

Thesis submitted to
National Institute of Technology, Rourkela
For award of the degree

of

Master of Technology

by

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Under the guidance of

Prof. Sandip Ghosh

DEPARTMENT OF ELECTRICAL ENGINEERING
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Dedication

To

My loving parents (Amma, Nanna)
My sisters (Annu, Veena, Aruna and Uma)

Declaration

I declare that the present thesis represents mostly my study and observations in my own words. Where other's ideas and work have been included, I have adequately cited and listed in the reference materials. I have adhered to all principles of academic honesty and integrity. No falsified or fabricated data have been presented in this thesis. I understand that any violation of the above will cause for disciplinary action by the Institute, including revoking the conferred degree, if conferred, and can also evoke penal action from the sources which have not been properly cited or from whom proper permission has not been taken.

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CERTIFICATE

This is to certify that the thesis entitled **A Comparative Study On Decoupling Methods For Time-Delay Systems**, submitted by **V Pavan Kumar** to National Institute of Technology, Rourkela, is a record of bonafide research work under my supervision and I consider it worthy of consideration for award of the degree of **Master of Technology** in **Electrical Engineering** with specialization in **Control and Automation** from this institute.

The embodiment of this thesis is not submitted in any other university and/or institute for the award of any degree/diploma to the best of our knowledge.

Prof. Sandip Ghosh

(Supervisor)

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V Pavan Kumar

Abstract

Decoupling of a multivariable system is much pronounced control problem in many industrial process. This has received much importance since last 20 years to develop decoupling controllers as an alternative to multi-loop PI controllers in order to achieve satisfactory set-point responses when there exist multiple time delays, non-minimum phase zeros and large uncertainties. In this thesis, static and dynamic decoupling control strategies are discussed.

Static decouplers are designed for control of multi-variable processes by RGA interaction analysis to determine the nature of interaction for static decouplers applied to multi-loop controllers and integral mode only at low and high frequencies. Here it is shown that static decouplers applied to integral modes perform better for processes in which non-diagonal elements decrease faster as frequency increases. This approach is sensitive to process changes and require detailed models.

In dynamic decoupling, internal model control and inverted decoupling methods are discussed here. Internal model control approach comprises both open-loop decoupling and closed-loop decoupling techniques. Internal model control is a centralized controller derived from the desired diagonal system matrix defined taking in consideration of time-delay compensation, non-minimum phase zeros and H_2 optimal performance objective; which effectively perform both decoupling and controlling functions. In open-loop IMC decoupling, the controller is designed from open-loop equation of process assuming that the model is exactly matching with the process. In closed-loop IMC decoupling, only model of the system is considered and controller is derived from closed-loop equation of the overall system. The presence of RHP zeros is elaborately discussed in different cases like no RHP zeros, finite RHP zeros, infinite RHP and LHP zeros and infinite RHP but finite LHP zeros. Inverted decoupling approach utilizes

inverse transfer matrix of the multivariable process alongwith dead-time compensator, when a FOPDT model of the process is considered. The decoupled process is controlled by using PI controllers. Wood Berry distillation column process is simulated for all these approaches. A comparison is made between these decoupling strategies when applied to the above process. The present work concludes by application of the above discussed decoupling strategies to the coupled tank system. The control of coupled tank liquid level system is a most challenging benchmark control problem due to its non-linear and non-minimum phase characteristics. The control objective in a coupled tank system is that a desired level of liquid in the two tanks is to be maintained independently when there is an inflow and outflow of water in the tanks respectively. The coupling effect here is the coupling switch which allows flow of water from a tank at high level to a tank at low level. Here, we have applied IMC closed-loop approach and inverted decoupling approach to the coupled tank system and then compared.

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List of Acronyms

MIMO	:	Multi Input Multi Output
SISO	:	Single Input Single Output
TITO	:	Two Input Two Output
PSUPA	:	Power Supply and Power Amplifier
TFM	:	Transfer Function Matrix
RGA	:	Relative Gain Array
ETF	:	Equivalent Transfer Function
SIMC	:	Simplified Internal Model Control
IMC	:	Internal Model Control
RHP	:	Right Half Plane
FOPDT	:	First Order Plus Dead-Time
SOPDT	:	Second Order Plus Dead-Time
DAQ	:	Data Acquisition

Chapter 1

Introduction

1.1 Overview

In process control, a term which is very often encountered by a control engineer is multivariable systems without which the characterization of any industrial process is incomplete. From the words itself multivariable system can be defined as a typical system which has several variables to be controlled. In process control, multi-variable systems are found in chemical reactors, heat exchangers and distillation columns, etc. However, the use of multivariable system is not limited to process control, but is well applicable in other control processes. Some of the examples which illustrate this are as follows:

- In air-cooling systems, temperature and humidity are difficult to control.
- A robot requires six degree-of-freedom to have full range of positioning.
- Missile tracking in military operations.
- Angle of elevation and speed control in an aeroplane.

1.2 Multivariable System Description

Depending upon the nature of multivariable systems (i.e., linear or non-linear), following representation of multivariable systems can be given as:

- State-space representation (for linear systems having p outputs and m inputs)

$$\begin{aligned} \dot{x}(t) =& Ax(t) + Bu(t) \\ y(t) =& Cx(t) + Du(t) \end{aligned} \tag{1.1}$$

where (A,B,C,D) are matrices of dimensions

$$A\epsilon R^{[n,n]}, B\epsilon R^{[n,m]}, C\epsilon R^{[p,n]} \text{ and } D^{R\epsilon[p,m]}$$

For simplicity, we remove 't' in further equations.

$$[\dot{x}] = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix}, [u] = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}, [y] = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix}$$

- Transfer function representation (for n variable system)

$$Y(s) = G(s)U(s)$$

$$\begin{bmatrix} y_1(s) \\ y_2(s) \\ \vdots \\ y_n(s) \end{bmatrix} = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{21}(s) & \cdots & g_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn}(s) \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \\ \vdots \\ u_n(s) \end{bmatrix}$$

where $G(s)$ is the transfer function matrix corresponding to appropriate input/output pairing.

1.3 Structure of Multivariable Models

Most of the multivariable systems can be decomposed into several two-by-two systems in engineering practice. The analysis of multivariable systems become much simpler for two-input-two-output (TITO) systems. There can be different structural representation of input/output models of multivariable system. The two most common structural forms are P-canonical form and V-canonical form [15]. By observing the above two canonical structures, it can

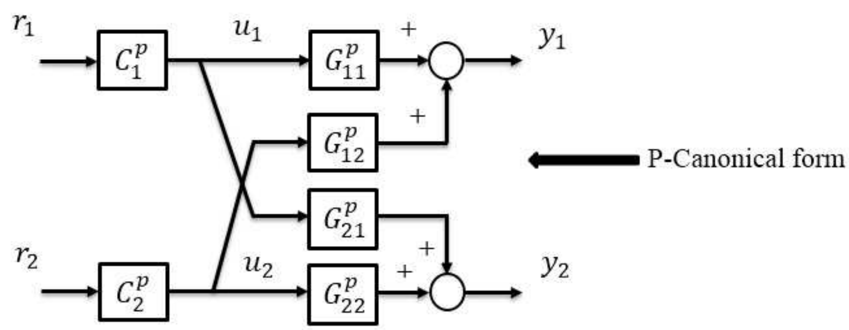


Figure 1.1: P-canonical form

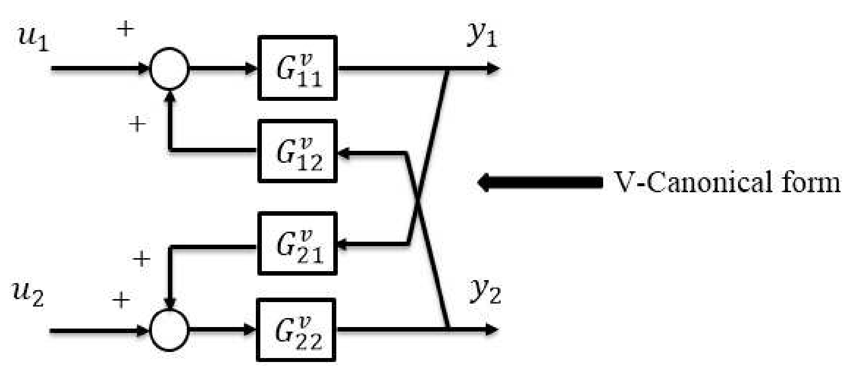


Figure 1.2: V-canonical form

be concluded that in P-canonical structure loop interactions are represented in feed-forward form whereas in V-canonical structure loop interactions are represented in feedback form. The elements $G_{11}^p, G_{12}^p, G_{11}^v, G_{12}^v$, *etc* are transfer functions related to corresponding input/output pair for both structural representations, respectively.

1.3.1 P-canonical Representation

The dynamics of P-canonical structure can be given by

$$\begin{aligned} y_1(s) &= G_{11}^p(s)u_1(s) + G_{12}^p(s)u_2(s) \\ y_2(s) &= G_{21}^p(s)u_1(s) + G_{22}^p(s)u_2(s) \end{aligned} \tag{1.2}$$

where $y_1(s)$ and $y_2(s)$ are the plant outputs and $u_1(s)$ and $u_2(s)$ are the manipulative plant inputs. In matrix-vector notation, the above relationship can be written as:

$$y(s) = G^p(s)u(s) \tag{1.3}$$

where

$$y(s) = [y_1(s)\ y_2(s)]^T, u = [u_1(s)\ u_2(s)]^T, G^p(s) = \begin{bmatrix} G_{11}^p(s) & G_{12}^p(s) \\ G_{21}^p(s) & G_{22}^p(s) \end{bmatrix}$$

1.3.2 V-canonical Representation

The dynamics of V-canonical structure can be given by

$$\begin{aligned} y_1(s) &= [y_2(s)G_{12}^v(s) + u_1(s)]\, G_{11}^v(s) \\ y_2(s) &= [y_1(s)G_{21}^v(s) + u_2(s)]\, G_{22}^v(s) \end{aligned} \tag{1.4}$$

In matrix-vector notation, the above relationship can be expressed as:

$$y(s) = [I - G_m^v(s)G_i^v(s)]^{-1}G_m^v(s)u(s) \tag{1.5}$$

where

$$G_m^v(s) = \begin{bmatrix} G_{11}^v(s) & 0 \\ 0 & G_{22}^v(s) \end{bmatrix} \text{ and } G_i^v(s) = \begin{bmatrix} 0 & G_{12}^v(s) \\ G_{21}^v(s) & 0 \end{bmatrix}$$

1.3.3 Relationship between P-canonical and V-canonical Representation

$$G^p(s) = [I - G_m^v(s)G_i^v(s)]^{-1}G_m^v(s)\,(\text{ provided that the inverse exist}) \tag{1.6}$$

An important consideration taken here is that 's' variable is excluded from all the transfer function representations for the further coming sections to avoid complexity.

1.3.4 Choice of System Representation

The system representation out of the two model forms should take the following factors into consideration:

- There should be possibility of determining the model parameters from experiments.
- The model must be characterizing the process, and preferably general enough to encompass other processes.
- The plant model should furnish necessary information required for controller design.
- It should possess simplicity.
- The processes are usually subject to external factors such as changes in the environment or in operating conditions. To account these factors, load disturbance should be incorporated in the model.

1.3.5 Advantage of P-canonical Form over V-canonical Form

- The elements of the transfer function matrices (G_m^v and G_i^v) in V-canonical representation cannot be obtained directly from open-loop step tests. They may be obtained from numerical identification techniques.
- The model in P-canonical representation is implicitly controllable and observable. In other words, the outputs are measurable and the inputs considered are relevant for control.

1.4 Characteristics of a Multivariable System

- A multivariable system has inherent cross-couplings or interaction between its variables.
- A multivariable system has different types of zeros such as system zeros, transmission zeros, input decoupling zeros and output decoupling zeros.
- It is possible in a multivariable system that pole and zero at same locations do not cancel each other.
- Multivariable processes are characterized by large uncertainties, time-delay and presence of non-minimum phase zeros.

These characteristics of a multivariable system inhibit from designing any of the loop independently as adjusting the controller parameters of one loop affects the other, sometimes lead to destabilizing the whole system. Thus, SISO(Single-Input-Single-Output) design techniques cannot be directly applied to multivariable processes because of the presence of interactions.

1.4.1 Statement of Interaction Problem

The impact of loop interactions on control system performance can be easily learned by taking an example.
Consider two independent first order, delay free processes G_{11} and G_{22} are under proportional control by K_{p11} and K_{p22} respectively. Then the characteristic equation of the two independent loops can be written as:

$$1 + K_{p11}G_{11} = 0 \, and \, 1 + K_{p22}G_{22} = 0$$

As we know a $1st$ order system utilizing proportional control hold stability at all times. Therefore, each loop will remain stable irrespective of the values of gains of the proportional controller. Let us now consider that there exists interaction between the two loops and the interaction dynamics are given by G_{12} and G_{21} as per P-canonical representation. The stability of the coupled system will now depend upon the characteristic equation:

$$(1 + K_{p11}G_{11})(1 + K_{p22}G_{22}) - G_{12}K_{p22}G_{21}K_{p11} = 0$$

This equation denotes that the system will remain stable only for a range of proportional gains. This may lead to system instability unless the interactions between the loops are considered in the control system design. The problem statement can be given as:
"If the steady state or dynamic gain of a given controlled variable in response to a given manipulated variable changes when other (initially open) loops are closed, then interaction exists in the system. If the controller in question was tuned with all others in manual, that tuning will be incorrect when all the others are placed in automatic because of their influence over the gain of that particular loop. Depending on the degree of interaction, instability or at the very least degraded responses will result."
Therefore, it is necessary to evaluate quantitatively the extent of interaction present between the control loops which can be used to structure a minimal control interaction scheme.

1.5 Control Strategies for Multivariable Systems

Depending upon the interaction persisting in the process, different control strategies can be applied to achieve desired responses. Following are the control strategies applied to multivariable systems:

- Centralized structure
- Decentralized structure
- Decoupled structure

1.5.1 Centralized Structure

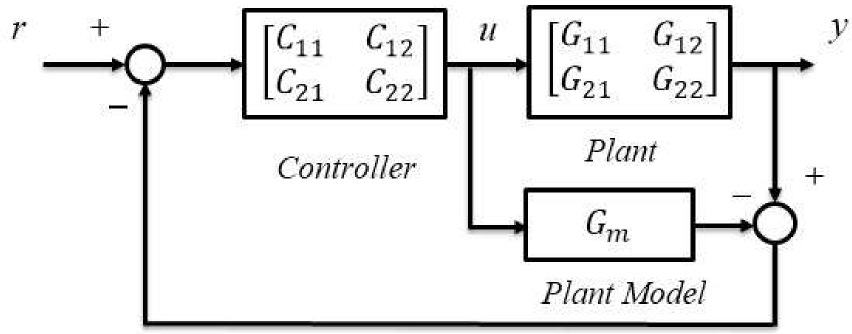


Figure 1.3: Centralized structure

- In this scheme, a full multivariable controller(nxn) is designed to control n output variables using n manipulated variables.
- The benefit of centralized controller is easy to tune even with the knowledge of the steady state gain matrix alone, multivariable PI controllers can be easily designed.
- The limitation of this scheme is the complexity in calculation of controller matrix and difficulty in understanding the control loops.
- For centralized structure, internal model control-proportional integral tuning can be used for tuning the controller.

1.5.2 Decentralized Structure

- In this scheme, the system is decomposed into a number of subsystems and individual controllers are designed for each subsystem.
- The advantage of decentralized controllers are:

  1. Easy to implement, easy to understand and easy to retune to account for change in process conditions.

  2. In case of failure of manipulated variables, tolerances can be easily incorporated into the design of decentralized controllers than full controllers.

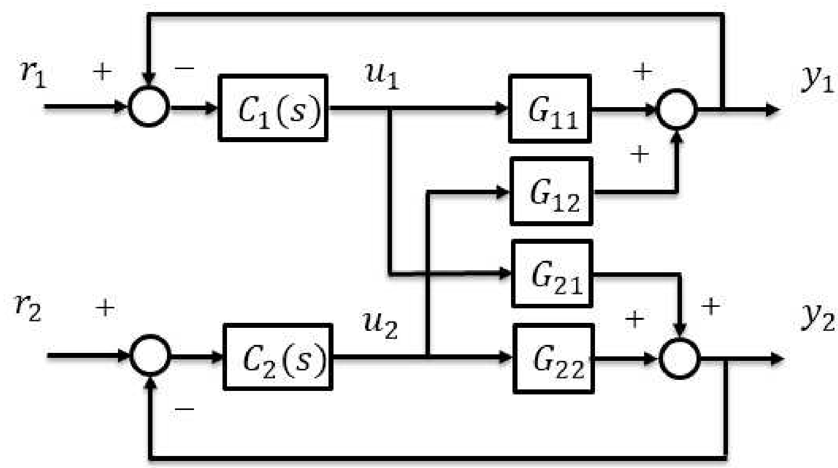


Figure 1.4: Decentralized structure

3. The control system can be brought gradually into service during process start up and taken gradually out of service during shut down.

4. When one controller goes off, it will not affect the other loop and stability is preserved.

- The limitation of this scheme is the interaction analysis required for pairing of input-output variables and the tuning of controllers is done through trial and error steps.

- Different methodologies for tuning of SISO controllers can be applied here(Ziegler-Nichols Criterion, Biggest log modulus tuning, Cohen-coon tuning, etc.).

1.5.3 Decoupled Structure

- This scheme utilizes separate elements known as decouplers to compensate for the strong interactions present in the system.

- To determine the nature of interaction present in the system interaction anlaysis (RGA analysis, Niederlinski index, singular value decomposition, Hankel index array, Dynamic RGA, etc) is required.

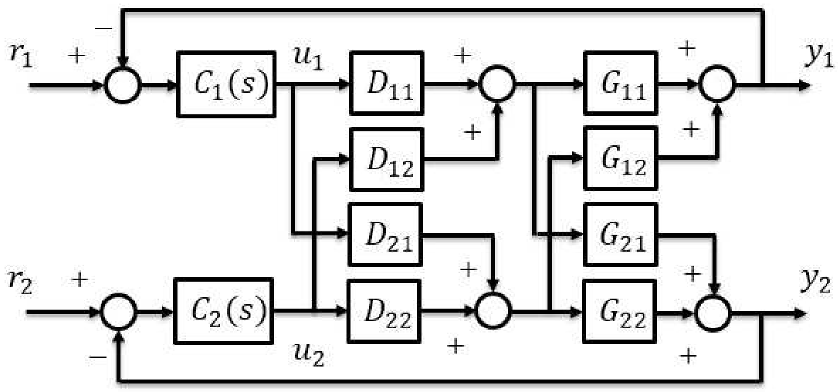


Figure 1.5: Decoupled structure

- In this scheme, the limitations of both centralized and decentralized structure are taken into account such as calculation complexity in centralized structure and pairing of input-output variables in decentralized structure.

1.5.4 Types of Decoupling

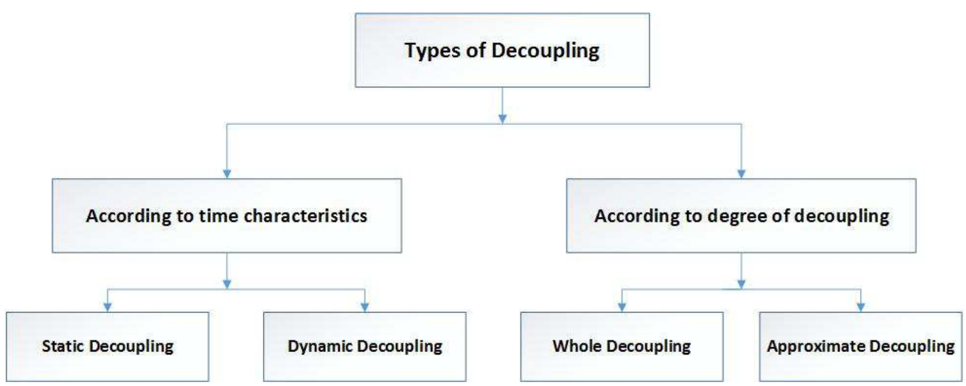


Figure 1.6: Types of decoupling

Decoupling can be divided into static and dynamic decoupling based on the time characteristics. Consider a system represented in state-space form as in 1.1. Assuming zero initial conditions ($i.e.x(0) = 0$) and input and output vectors to be of same dimension m, the output equation related by transfer function matrix given as:

$$y(s) = G(s)u(s) = C(sI - A)^{-1}Bu(s)$$

The above equation can be expressed as:

$$y_1(s) = g_{11}(s)u_1(s) + g_{12}(s)u_2(s) + \cdots\cdots + g_{1m}(s)u_m(s)$$

$$y_2(s) = g_{21}(s)u_1(s) + g_{22}(s)u_2(s) + \cdots\cdots + g_{2m}(s)u_m(s)$$

$$\vdots$$

$$y_m(s) = g_{m1}(s)u_1(s) + g_{m2}(s)u_2(s) + \cdots\cdots + g_{mm}(s)u_m(s)$$

A clear observation from above equation is that these equations are coupled, since each individual input affect all the outputs. It is necessary to determine appropriate control inputs $u_1, u_2, \cdots\cdots, u_m$ such each input control only the corresponding output without affecting any of the other outputs.

- **Static Decoupling:** A system of form $y = G(s)u(s) = C(sI - A)^{-1}Bu(s)$ is said to be statically decoupled if it is stable and its static gain matrix $G(0)$ is diagonal and non-singular. Consider for example a step input $u(t) = \alpha$, where $\alpha = [\alpha_1, \alpha_2, \alpha_3 \cdots \alpha_m]^T$ is constant, then the outputs should satisfy

$$lim_{t\to\infty} y_1(t) = g_{11}\alpha_1$$

$$lim_{t\to\infty} y_2(t) = g_{22}\alpha_2$$

$$\vdots$$

$$lim_{t\to\infty} y_m(t) = g_{mm}\alpha_m$$

- **Dynamic Decoupling:** A system of form $y = G(s)u(s) = C(sI - A)^{-1}Bu(s)$ is said to be dynamically decoupled if it is stable and its transfer function matrix $G(s)$ is diagonal and non-singular.

$$y_1(s) = g_{11}(s)u_1(s)$$

$$y_2(s) = g_{22}(s)u_2(s)$$

$$\vdots$$

$$y_m(s) = g_{mm}(s)u_m(s)$$

where $g_{ii}(s) \neq 0, i = 1, 2, .., m$

In dynamic decoupling, the multivariable system can be considered as consisting of m independent subsystems. Dynamic decoupling is complex as it requires a highly sensitive control law.

Decoupling can be categorized into whole and approximate decoupling based on the degree of decoupling.

- **Whole Decoupling:** In this type of decoupling, the multivariable system is said to be ideally decoupled such that there exists no more interaction in the system and the multivariable system can be represented into several independent sub-systems which can be controlled by employing SISO control techniques.

- **Approximate Decoupling:** In practical scenario, it is very difficult to achieve whole decoupling. This is because of the presence of uncertainties in process models as well as the disturbances occurred during process operation. Another cause is the parameterization of the process which limits to remove the coupling effect completely. This type of decoupling is called approximate decoupling.

1.5.5 Importance of Decoupling Control

- The characteristic properties of a multivariable system (i.e. large uncertainty, strong coupling, large time delay, presence of non-minimum phase zeros and nonlinearity) inhibits the performance and robustness of a multivariable system.

- To compensate for the interactions present in the system as far as possible so that a specific output can be controlled by a specific input. As MIMO system is to be converted into several independent SISO loops, the decoupling control demands proper input-output pairing which requires interaction analysis.
- A very need of decoupled systems is to accomplish online tuning of the closed loop system according to the needs of control or change in conditions; although the controller cannot be tuned online due to existence of interaction between the variables.
- The variables present in the MIMO system cannot be considered individually when there is significant interaction present in the system. So, there is a need to adjust these variables in real time to obtain acceptable results. However, it cannot meet the requirement.

1.6 Literature Review

Prior to 1940's most industrial systems were controlled manually or by using on-off control. Several operators were required to keep an eye on the different variables present in the system. The advent of automatic control appeared as a solution to the control of physical variables in order to meet the demand of high precision, quality and efficiency in lieu of increased labour and equipment costs. Today automatic controllers are widely used in many industries successfully. As the industrial processes have several variables to be controlled, these systems were called multivariable systems and the control theory dealing with these systems was known to be as decoupling control.

The study of decoupling control has received considerable attention in both control theory and industrial practices dating back to 1950s. In [16], the decoupling control problem was treated with transfer matrices. In 1964, [13] proposed state space approach to formulate the problem of decoupling a multivariable system into single input single output subsystems. The more defined block decoupling problem was solved using geometric approach as given in [4]. It made possible to formulate the problem of decoupling a general linear system into arbitrary size subsystems, giving solutions for a number of cases. These methods were mainly confined to square systems only. In later 90s, the complete solution to the generalised Morgan problem was given by Descusse et al [3].

A parallel study of decoupling control problem was going on for unity feedback systems based on input-output models. The existing conditions for control of such processes were that the plant has full row rank, simple and well known. The problem of decoupling was to maintain internal stability of the decoupled system because unstable outputs force actuators to wear out. Diagonal decoupling control scheme was proposed involving parameterization of controllers under the assumption of plant having no unstable poles coinciding with its zeros [7]. This assumption was later relaxed by Wang et al [17] by proposing a decoupling method for 2×2 processes along with a controller parameterization. The results of Wang were generalized t block decoupling problems for non-square plants and performance was compared with that of non-decoupled systems as given in [8].

The decoupling schemes discussed above were confined to plants without time delay which was a very impractical assumption. However, time delay exists in every real time process and therefore the applicability of above developed theories in practical scenario was unsuccessful as time delay posed difficulty in process operation and control. It prevents high gain of the controller from being used, leading to offset and sluggish response. When we talk about multivariable systems, the condition becomes worse there are multiple delays existing in the system. Smith suggested a compensation scheme to remove time delay from the characteristic equation thus easing feedback control design applicable for SISO plants with time delay. This was extended by some researchers in mid $90s$, but the design of controller was difficult to carry out and it failed to achieve the nominal performance from the delay-free design.

This problem was later solved by [18] in which a robust system identification method from step tests is presented first to SISO systems and then extended to TITO (Two-Input-Two-Output) processes. A lead-lag controller is designed from the FOPDT (First Order Plus Dead Time) model of the process. Simplified decoupler design is used and further modification is done in decoupler to prevent the decoupler elements from being unstable. The decentralized PID controllers are designed using frequency domain approach and auto-tuning steps are described. [2] Model predictive control operate in supervisory mode with sampling times longer than in PID loops. This make difficulties in dealing with interactions when lower level PID loops are closed.

Interaction analysis is an important issue in control structure design which can be achieved by RGA (Relative Gain Array) analysis. A static decoupler with modified PI design is proposed considering coupling between the lower loops. Lee et al performed RGA analysis for static decoupler applied to multi-loop controller and only to integral mode of PI controller. The analysis concludes that the interactions become worse at high frequency when static decouplers are applied to multi-loop controller. [6] The design of PI controller is done by using taylor series approximation. An anti-reset windup scheme guaranteeing control performances under process saturation failures is provided.

In [10] a novel solution is provided for the limiting interactions present in decentralized control of MIMO (Multi-Input-Multi-Output) systems. In this approach, the reference vector components are tuned to avoid the limits imposed by the control designer on the interactions. This reduces the coupling effects and ensures a safer operation of MIMO systems. This approach is even applicable to systems with non-minimum phase zeros in their diagonal elements. Due to presence of time delays and non-minimum phase zeros the controller tuning aims at the compromise between achievable system performance and decoupling regulation. Also, there exist uncertainties present in the system which needs a control action to be robust. To achieve this an analytical method based on H_2 optimal performance objective is proposed in (12, 2006 liu), which results in in ideal desired decoupling controller matrix to be derived within the framework of unity feedback control structure. This approach is called internal model control. Pade approximation is utilized to implement the practical form of controller. The results were compared with (8,9,10) concluding better performance and decoupling regulation. In [11], a more elaborative IMC controller design is proposed for different cases of non-minimum phase zeros such as no RHP (Right Half Plane) zero, finite RHP zeros, infinite RHP and finite LHP zeros, infinite RHP and LHP zeros. Robustness analysis is defined for three types of uncertainties namely process additive, process multiplicative input and output uncertainty which is analysed using spectral radius criterion.

Decentralized Decoupling methods are mainly of three types ideal, simplified and inverted. Ideal decoupling utilizes the inverse of transfer function (TFM) matrix of the process. The realizability of the inverse of TFM is a critical issue as we have proper transfer functions of the process. Another problem with ideal decoupling is that it is sensitive to modelling errors. Simplified decoupling uses a simple decoupler form, but it requires model reduction before implementation. Inverted decoupling has the advantage of both ideal and simplified decoupling and is more suitable for implementation. In [19], an analytical operation along with a definite search algorithm is proposed to design an effective dead time compensation matrix. After then the conventional decoupling is applied to the compensated process and an IMC based PID decentralized controller is designed to achieve better set-point response and disturbance rejection.

Decentralized PI/PID controllers are designed based on the equivalent transfer function (ETF) model of the individual loops and simplified decoupler is proposed in [14]. The equivalent transfer function is obtained by relative normalized gain array and relative average residence time array analysis. By using dynamic RGA analysis, it is shown that effective open-loop transfer function is equivalent to equivalent transfer function model. The decentralized PI controllers are designed using the simplified internal model control (SIMC) method. The performance of proposed method is compared with ideal, normalized, inverted decoupling and centralized control structure.

In [12], some model based strategies applied quadruple tank process are compared. The strategies are decentralized control, decoupling control and IMC control. In decentralized control, first interaction measurement is done by RGA analysis in order to check proper input-output pairing. Then condition number was computed to analyse if the diagonal matrix is easy or difficult to control than the full matrix. The results of decentralized control reveals that there is a need of decoupler to take into account the interaction between the loops. A dynamic decoupler is designed for this purpose. Finally an IMC controller is designed which gives better desired set-point trajectory and disturbance rejection.

A review of decoupling control methods was presented in [9] based on multiple models. The adaptive decoupling controllers are designed by attempting, training or optimization based on traditional decoupling methods such as diagonal matrix synthesis, characteristic locus analysis, inverse nyquist array, singular value decomposition, etc. However, the theory proposed here is not perfect and requires further investigation about its combination with multivariable decoupling technology and industrial process.

1.7 Motivation

With the advancement of technology in industries, the demand of getting a desired output with utmost efficiency becomes very high. Most of the industries are dealing with multivariable or multiple-input-multiple-output systems. Interaction between these variables is a general characteristic which is associated with other system properties like time-delay and non-minimum phase zeros. These characteristics limit the closed loop performance. This effect can be seen as when loop gain increases, poles move toward zeros and thus destabilization inevitably occurs which limits the bandwidth of the closed loop system. In such cases, unstable pole-zero cancellation is not an allowable operation in practical controller as an arbitrary small discrepancy between zero and pole results in instability. The controller output to the process becomes unbounded enhancing the chances of actuators to wear out in practical scenario. Therefore, the assessment of the nature of interaction present in different processes is a very important consideration in designing controllers as it varies from process to process randomly. If somehow we do interaction analysis and determine whether there is requirement of a decoupler whether static or dynamic depends upon our performance specifications. However, when there is significant interaction present in the system, it becomes a challenging task to design decouplers as the multi-loop PI/PID controllers are unable to give satisfactory results. This motivates us to study the different decoupling strategies with its application to real time systems.

1.8 Objectives

The present thesis deals with the following objectives:

- To study static and dynamic decoupling techniques for control of two-input-two-output (TITO) systems and to compare the performance of these decoupling techniques applied to wood berry distillation column process model.

- To study coupled tank model and compare the performance of IMC and inverted decoupling controllers applied coupled tank system in real time.

1.9 Organization of the Thesis

- Chapter 2 deals with static decoupling. A static decoupler is designed for a wood berry distillation column process.
- Chapter 3 describes dynamic decoupling. In this, three dynamic decoupling strategies are discussed and then applied to design decoupling controller for Wood Berry distillation column process.
- Chapter 4 gives a brief introduction of coupled tank system. The model of coupled tank is determined and decoupling controllers are designed to analyse the practical applicability of the decoupling strategies.
- Chapter 5 concludes the thesis. Future scope of decoupling is discussed.

Chapter 2

Static Decoupling

This chapter briefly expresses the static decoupling requirements and interaction analysis for a TITO(Two-Input-Two-Output) process. A static decoupler is designed alongwith a PI controller based on multi-loop IMC design strategy for Wood Berry distillation column process.

2.1 Introduction

Decouplers are used when performance of multi-loop control systems is degraded due to interactions present in the system. Static decouplers are used when fast controls are not required in a system. They possess simple simple structure and their design does not require detailed description of the system; which is difficult to obtain. In general, static decouplers are designed from steady-state gains which can be easily obtained and tuned in the field. These decouplers are straight forward extension of single-input-single-output (SISO) proportional integral (PI) controllers to multi-input-multi-output(MIMO) systems. In literature, a multivariable PI/PID controller has been designed by using Taylor series expansion technique which was proposed by Tan et. al. A full rank controller matrix is obtained from detailed model of the system which is difficult to tune [6].

2.2 Problem Definition

Consider a process of n inputs and n outputs whose transfer function is:

$$G(s) = \{g_{kl}(s), k = 1, ..., n, l = 1, ..., n\}$$

The controller structure for the static decoupler on both proportional-integral mode can be represented as:

$$\begin{aligned} C(s) =& G^{-1}(0)(\frac{1}{s}K_i + K_c) = \frac{1}{s}G^{-1}(0)\begin{bmatrix} K_{i1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & K_{in} \end{bmatrix} \\ &+ G^{-1}(0)\begin{bmatrix} K_{p1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & K_{pn} \end{bmatrix} \end{aligned} \tag{2.1}$$

$$C(s) = \frac{1}{s}G^{-1}(0)K_i + K_p = \frac{1}{s}G^{-1}(0)\begin{bmatrix} K_{i1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & K_{in} \end{bmatrix} + \begin{bmatrix} K_{p1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & K_{pn} \end{bmatrix} \tag{2.2}$$

The overall system structure will look like as follows:

The problem statement is static decouplers when applied to whole PI controller introduce

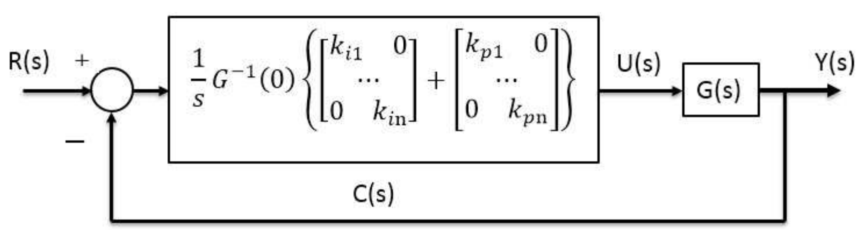


Figure 2.1: Static decoupler applied to whole PI controller

pronounced interactions at high frequencies for some processes. In such processes, the magnitude of off-diagonal elements decay faster than those of diagonal elements with the increase in frequency. Thus, interactions decrease as we increase frequency. When static decouplers are

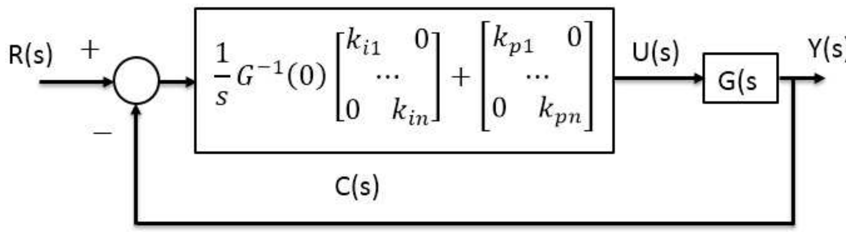


Figure 2.2: Static decoupler applied to integral mode of PI controller

applied to such processes; the process interactions are reduced at frequencies near zero. But this introduces undesirable effects at high frequencies as the magnitude of off-diagonal elements will not reduce enhancing process interactions at high frequencies. This can be justified by relative gain array analysis.

2.3 Static Decoupler Design

The decoupler design includes two steps; one is Relative Gain Array (RGA) analysis and other is design of PI controller.

2.3.1 RGA analysis

Relative Gain Array is a normalized form of the gain matrix that describes the impact of each control variable on the output, relative to each control variable impact on other variables. The relative gain array of a plant having transfer function $G(j\omega)$ can be calculated by

$$R[G(j\omega)] = G(j\omega)^{-T}.G(j\omega)$$

where . represents element-by-element multiplication. By using this definition, relative gain array is computed for static decouplers applied to whole PI controller as well as static decouplers applied to only integral mode of PI controller. The process considered here is wood berry distillation column whose model transfer function is given by:

$$G(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{(16.7s+1)} & \dfrac{-18.9e^{-3s}}{(21s+1)} \\ \dfrac{6.6e^{-7s}}{(21s+1)} & \dfrac{-19.4e^{-3s}}{(14.4s+1)} \end{bmatrix} \tag{2.3}$$

The observations made from relative gain array analysis are [6]:

- The $r_{11}(G(s)G^{-1}(0))$ of RGA at frequencies near to zero is around one and the interaction is removed by static decoupler.
- The $r_{11}(G(s)G^{-1}(0))$ of RGA increases to a large value as the frequency is increased.

From above observations, one can conclude that process interactions become worse when static decoupler is applied to whole PI controller. This will increase sensitivity of the system towards process uncertainties. To avoid this, we prefer to apply static decoupler to integral mode of PI controller.

2.3.2 PI Controller Design

The closed loop response of the system shown in 2.2 can be given by:

$$Y(s) = H(s)R(s) = (I + G(s)C(s))^{-1}G(s)C(s) \tag{2.4}$$

where

$G(s) \rightarrow$ plant transfer function

$C(s) \rightarrow$ controller transfer function

$H(s) \rightarrow$ Closed loop transfer function

$R(s) \rightarrow$ Set-point

$Y(s) \rightarrow$ Controlled variable

The PI controller is designed using multi-loop internal model controller design strategy. According to this strategy, each element of the desired closed loop response is given by:

$$h_{ii}(s) = diag([I + G(s)C(s)]^{-1}G(s)C(s))diag\left(\frac{{g_{ii}}^{+}(s)}{(\alpha_i s+1)^{n_i}}\right) \tag{2.5}$$

where ${g_{ii}}^+(s)$ and n_i are the non-minimum phase part and relative order of $g_{ii}(s)$ respectively and α_i is the tuning parameter representing the closed loop time constant. A very important assumption taken here is ${g_{ii}}^+(0) = 1$ such that the set-point can be tracked.

Let the desired closed loop response R(s) be given by

$$h_{ii}(s) = diag[h_1, h_2, ..., h_n] \tag{2.6}$$

Now our aim is to define C(s) such that all the diagonal elements of H(s) resemble those of R(s) as close as possible over a frequency range relevant to control applications.

$c_{ii}(s)$ can be expressed in Maclaurin series as:

$$c_{ii}(s) = \frac{k_{0i}}{s} + k_{1i} + k_{2i} + \cdots = \frac{1}{s}(k_{0i} + k_{1i}s + \cdots) \tag{2.7}$$

where k_0, k_1 corresponds to integral and proportional terms of the multi-loop PI controller, respectively. $c_{ii}(s)$ can be expressed in terms of $h_{ii}(s)$ as:

We know $h_{ii}(s) = \dfrac{g_{ii}(s)c_{ii}(s)}{1 + g_{ii}(s)c_{ii}(s)}$

$$c_{ii}(s) = \frac{1}{g_{ii}(s)} \frac{h_{ii}(s)}{1 - h_{ii}(s)} \tag{2.8}$$

By adjusting the integral part to compensate for the gain change resulting from closed loops and truncating 2.7, we obtain the multi-loop controller as:

$$C(s) = \frac{1}{s}K_i + K_p = \frac{1}{s}diag(k_{0i}g_{ii}(0)[G^{-1}(0)]_{ii}) + diag(k_{1i}) \tag{2.9}$$

The integral part is dominant at low frequencies, this approximates 2.5 well at low frequencies. The proportional part is not adjusted because it is dominant at high frequencies and we can approximate H(s) as:

$$H(s) = [I + G(s)C(s)]^{-1}G(s)C(s)$$

given that $\|G(s)C(s)\| \leq 1$ at high frequencies.

$$C(s) = \frac{1}{s}G^{-1}(0)K_I + K_P = \frac{1}{s}G^{-1}(0)diag[k_{0i}g_{ii}(0)] + K_P \tag{2.10}$$

where $K_I = diag[k_{0i}g_{ii}(0)]$ and $K_P = K_p$

These controller matrices only depend on the diagonal elements of G(s). Thus, they need not to be redesigned, where some input-output pairs are removed.

2.4 Design Example

Consider the Wood-Berry distillation column process as defined in 2.3

$$G(s) = \begin{bmatrix} \frac{12.8e^{-s}}{(16.7s+1)} & \frac{-18.9e^{-3s}}{(21s+1)} \\ \frac{6.6e^{-7s}}{(21s+1)} & \frac{-19.4e^{-3s}}{(14.4s+1)} \end{bmatrix}$$

2.4.1 Design Steps

Step1: Determine $G^{-1}(0)$ i.e. inverse of G(0)

$$G^{-1}(0) = \begin{bmatrix} 0.157 & -0.1529 \\ 0.0534 & -0.1036 \end{bmatrix}$$

Step2: Define $h_{11}(s)$ and $h_{22}(s)$. The exponential term will behave as a right half part of G(s) when linear approximation of delay is considered. Choose $\alpha_i = 5$

As per definition given in 2.5

$$h_{11}(s) = \Big(\frac{e^{-s}}{(5s+1)}\Big)$$

$$h_{22}(s) = \Big(\frac{e^{-3s}}{(5s+1)} Big)$$

Step3: Define $c_{11}(s)$ and $c_{22}(s)$ as per definition given in 2.8

$$c_{11}(s) = \Big(\frac{(16.7s+1)}{12.8*(5s+1-e^{-s})}\Big)$$

$$c_{22}(s) = \Big(\frac{-(14.4s+1)}{19.4(5s+1-e^{-3s})}\Big)$$

Step4: Use Maclaurin series expansion of $sc_{ii}(s)$ to determine the elements of PI controller.

$$k_{i1} = 0.013, k_{p1} = 0.2185, k_{i2} = -0.0064, k_{p2} = -0.0964$$

Step5: Substituting the elements of PI controller in the controller matrix form 2.10, we get

$$C(s) = \frac{1}{s}\begin{bmatrix} 0.0261 & -0.019 \\ 0.0089 & -0.0129 \end{bmatrix} + \begin{bmatrix} 0.2185 & 0 \\ 0 & -0.0964 \end{bmatrix}$$

2.5 Results and Discussions

The simulation results of static decoupler applied to wood berry distillation column process are as follows: The following observations can be drawn from above plots.

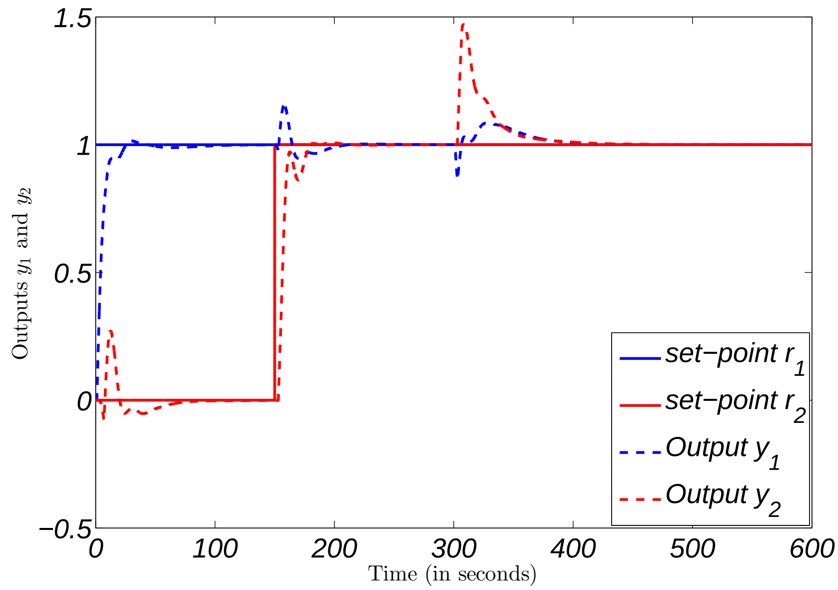


Figure 2.3: Nominal system responses

- The reference input r_1 and r_2 are applied at t=0 and t=150 seconds, respectively. The statically decoupled plant of wood berry distillation column process is capable to track their corresponding reference inputs. The decoupler effectively decouple the outputs in steady state and also upto some extent in transient state.

- It can be easily observed from 2.3 that there is some oscillation present in both the outputs y_1 and y_2 during transient period. This is due to the presence of interaction at high frequencies. After a certain time of around 100 seconds, these oscillations die out.

- To check the feasibility of the static decoupler approach, an inverse load disturbance of magnitude 0.1 is applied to both the process inputs at 300 seconds. It can be observed from the figure that both the outputs y_1 and y_2 are able to restore at their steady state values after a short time period. This time depends upon the magnitude of disturbance as well as the decoupling capability of controller.

- In addition to load disturbances, a very desired feature of every controller is robust stability. This tells whether the controller is robust enough to perform its objective of decoupling the system when there exists uncertainties in the model of the system. These uncertainties may due to improper modeling or due to changes in parameters of the system with respect to environmental conditions.

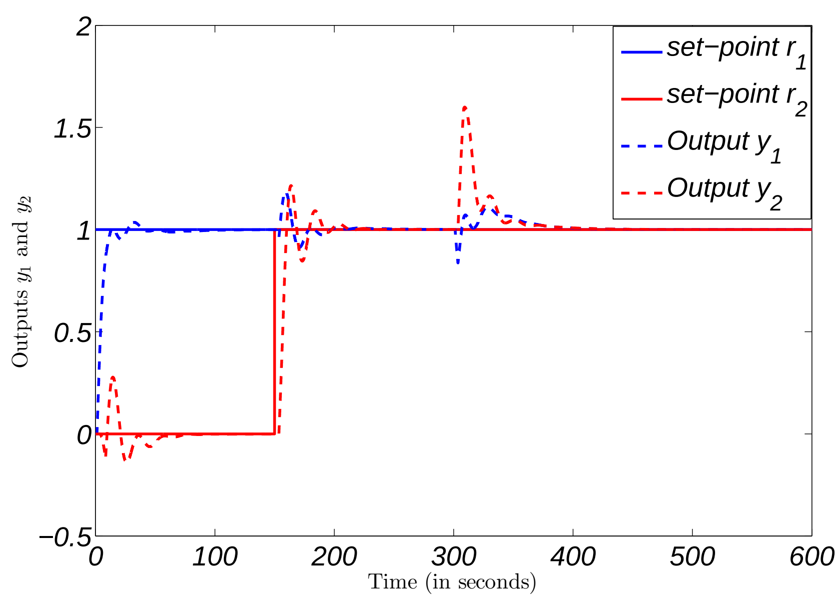


Figure 2.4: Perturbed system responses

- To check robust stability, we have introduced here additive uncertainties in the plant model and observed the changes in plant outputs y_1 and y_2. From 2.4, it can be said that this approach fails to provide a fair degree of robustness towards process uncertainties. The oscillations in outputs y_1 and y_2 increase in transient period due to changes in parameters of the plant. At the time of load disturbances, the variation in outputs y_1 and y_2 increase; which further increases the time required to regain its tracking path.

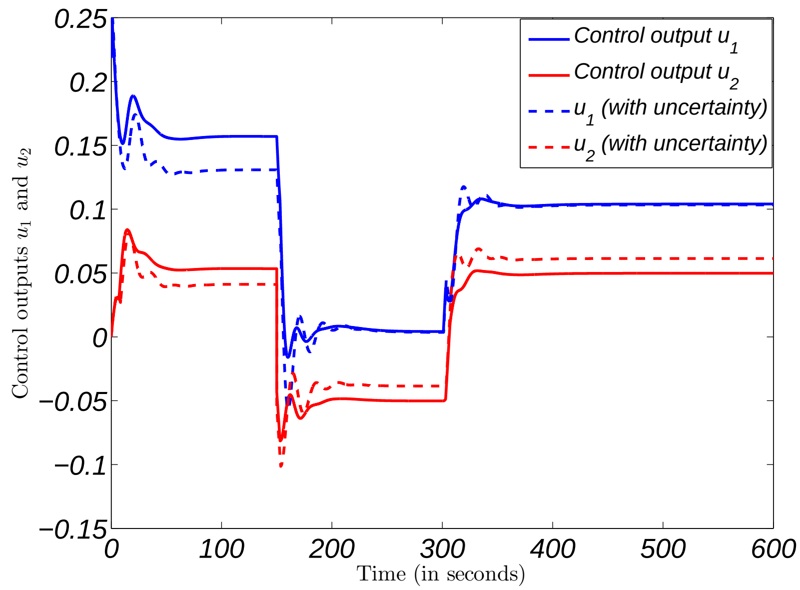


Figure 2.5: Control outputs for nominal and perturbed system

- The control outputs u_1 and u_2 in both cases (i.e in presence and absence of uncertainties) are within the range of 0.25 to -0.25. This justifies its applicability in real time as the actuators present in practical scenario work well for this range of variations in control outputs.

- From all these observations, one can conclude that static decoupler is suitable to achieve decoupling in steady state whereas it is difficult to effectively decouple the system in transient period. in addition to this, robust stability is not good enough to maintain the decoupling feature efficiently.

Chapter 3

Dynamic Decoupling

This chapter gives a detailed analysis of different dynamic decoupling technique applied to multivariable systems. The comparison of these decoupling techniques is analysed by its application to Wood Berry distillation column process.

3.1 Introduction

Interactions deteriorate the performance required from a multivariable system. Dynamic decouplers are used where coupling between the plant variables is strong throughout the range of operating frequency. The very requirement of dynamic decoupling is detailed information about plant models. Unlike static decouplers which are based on static gains, dynamic decouplers are based on rational functions of inverse transfer matrix. There can be several approaches to dynamic decoupling such as ideal decoupling, simplified decoupling, inverted decoupling, internal model control decoupling, state space approach, etc. The approaches discussed in present work are Internal Model Control (IMC) approach I, Internal Model Control (IMC) approach II and Inverted Decoupling approach.

3.2 Problem Definition

The general control structure for a MIMO (Multi-Input-Multi-Output) system is shown below.

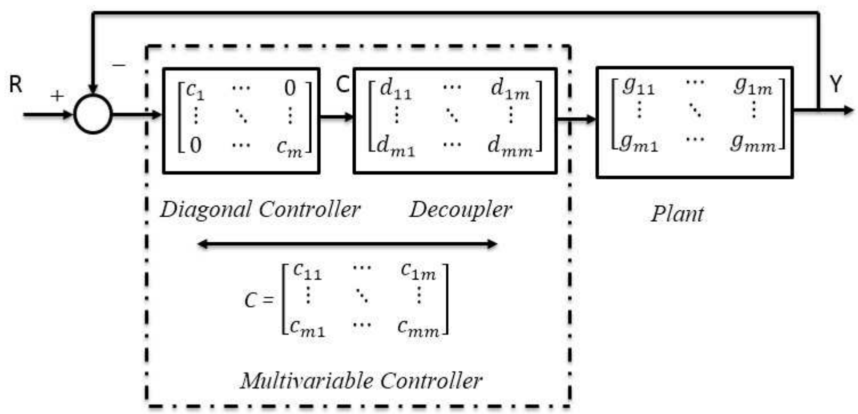


Figure 3.1: General dynamic decoupling structure

The problem is to design a multivariable controller C(s) which can decouple the MIMO system shown in figure above. The function of multivariable controller is both decoupling and controlling action. Another way of designing a controller for above MIMO system is a decoupler which can convert the system into independent subsystems and then a diagonal controller which can be designed using any SISO (Single-Input-Single-Output) design techniques.

3.3 Internal Model Control Approach I

Internal Model Control (IMC) refers to a systematic procedure for control system design based on internal model principle that is the basis for many modern control techniques.

Internal Model Principle - Any good regulator must create a model of the dynamic structure of the environment in the closed loop system or Any good tracking controller must stabilize the closed loop system and must contain a model of the reference.

Consider a general TITO (Two Input Two Output) plant with time delays as given below [10]

$$G(s) = \begin{bmatrix} \dfrac{k_{11}e^{-\theta_{11}s}}{\tau_{11}s+1} & \dfrac{k_{12}e^{-\theta_{12}s}}{\tau_{12}s+1} \\ \dfrac{k_{21}e^{-\theta_{21}s}}{\tau_{21}s+1} & \dfrac{k_{22}e^{-\theta_{22}s}}{\tau_{22}s+1} \end{bmatrix} \tag{3.1}$$

The genaralized IMC structure for a TITO plant is shown below [10]

where G_m is the process model. The system transfer matrix considering presence of uncertainty

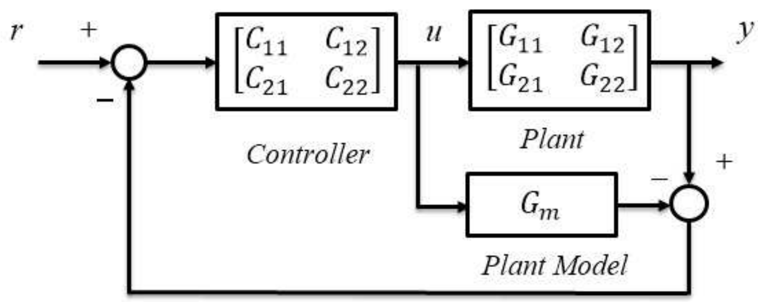


Figure 3.2: General IMC structure

in the plant parameters i.e., unmodeled dynamics of the plant is of the form given below:

$$H = GC[I + (G - G_m)\,C]^{-1} \tag{3.2}$$

The above equation will introduce complexity and there are chances of losing stability. In that case the controller designed for the nominal plant model may not ensure robust stability of the system any longer. To solve this problem, the controller is tuned analytically as per reference plant model in a simple manner in order to achieve desired nominal system responses alongwith decoupling regulation. The decoupling controller should be tuned online to cope with the unmodeled dynamics of the plant. When $G = G_m$, the above system will behave as open loop in absence of any external(load) disturbances. For such system, the transfer matrix will look like as below:

$$H = GC = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \tag{3.3}$$

The decoupled system responses for a nominal TITO(Two Input Two Output) plant (when G=G_m) should correspond to a diagonal system transfer matrix,i.e.,

$$H = \begin{bmatrix} h_{11} & 0 \\ 0 & h_{22} \end{bmatrix} \tag{3.4}$$

where h_{11} and h_{22} need to be stable and in proper form. To accomplish this, two decoupling preconditions need to assured as follows:

- Both plant and controller are required to be non-singular at $s = 0$. This means determinant of plant and controller matrix in steady state must not be equal to zero, i.e., $det\left[G\left(0\right) \neq 0\right]$ and $det\left[C\left(0\right) \neq 0\right]$.

- There should not be any cross-coupling in tuning each column controller of C.

The controller matrix C can be directly derived from 3.4 as:

$$C = G^{-1}H = \frac{adj\left(G\right)}{det\left(G\right)}H \tag{3.5}$$

where $adj\left(G\right) = \left[G^{ij}\right]^{T}{}_{2\times 2}$ is the adjoint of G matrix and G^{ij} represents minor corresponding to g^{ij} in $G, i, j = 1, 2$. The determinant of plant G can be expressed as:

$$det(G) = \begin{cases} G^{11}G^{22}\left(1 - G^{\circ}e^{-\Delta\theta s}\right) for\left(\theta_{11} + \theta_{22} \leq \theta_{12} + \theta_{21}\right) \\ -G^{12}G^{21}\left(1 - \dfrac{e^{-\Delta\theta s}}{G^{\circ}}\right) for\left(\theta_{11} + \theta_{22} \geq \theta_{12} + \theta_{21}\right) \end{cases} \tag{3.6}$$

where

$$\Delta\theta = \|\theta_{11} + \theta_{22} - \theta_{12} - \theta_{21}\|$$

$$G^{\circ} = \frac{k_{12}k_{21}}{k_{11}k_{22}} \cdot \frac{\left(\tau_{11}s + 1\right)\left(\tau_{22}s + 1\right)}{\left(\tau_{12}s + 1\right)\left(\tau_{21}s + 1\right)}$$

The above definition of the determinant of the process transfer matrix gives rise to two cases. Firstly, consider the case where $\theta_{11} + \theta_{22} \leq \theta_{12} + \theta_{21}$ of some TITO processes. The first column of controllers can be written as:

$$c_{11} = \frac{G^{11}}{det\left(G\right)} h_{11} = \frac{1}{G^{22}\left(1 - G^{\circ} e^{-\Delta\theta s}\right)} h_{11} = \frac{\left(\tau_{11} s + 1\right) e^{\theta_{11} s}}{k_{11}\left(1 - G^{\circ} e^{-\Delta\theta s}\right)} h_{11} \tag{3.7}$$

$$\begin{aligned} c_{21} =& \frac{G^{12}}{det\left(G\right)} h_{11} = \frac{G^{12}}{G^{22} G^{11}\left(1 - G^{\circ} e^{-\Delta\theta s}\right)} h_{11} \\ =& -\frac{k_{21}\left(\tau_{11} s + 1\right)\left(\tau_{22} s + 1\right) e^{\left(\theta_{11} + \theta_{22} - \theta_{21}\right) s}}{k_{11} k_{22}\left(\tau_{21} s + 1\right)\left(1 - G^{\circ} e^{-\Delta\theta s}\right)} h_{11} \end{aligned} \tag{3.8}$$

From 3.7 and 3.8, it is observed that controller matrix elements c_{11} and c_{21} are related to h_{11} element of H matrix. This means controller elements in one column will depend on its corresponding diagonal element of desired closed loop transfer function matrix. If h_{11} does not include an equivalent time delay to offset θ_{11} , the controller behaviour can be predicted inevitably and so does c_{21} if $\theta_{11} + \theta_{22} \leq \theta_{21}$, which is not possible because either of the plant outputs can start to track its corresponding set-point only after certain time delay. If the polynomial $\left(1 - G^{\circ} e^{-\Delta\theta s}\right)$ in 3.7 and 3.8 contain any RHP zero, then it is required that h_{11} will include these RHP zeros such that c_{11} and c_{21} will not bundle them as unstable poles.

The desired diagonal transfer function can be defined on the basis of presence of right half plane zeros of $\left(1 - G^{\circ} e^{-\Delta\theta s}\right)$ and robust H_2 performance objective.

$$h_{11} = \frac{e^{-\theta_1 s}}{\alpha_1 s + 1} \prod_{i=1}^{n} \left(\frac{-s + z_i}{s + z_i^{\star}}\right) \tag{3.9}$$

where $\alpha_1 \rightarrow$ adjustable tuning parameter used for meeting system response requirement for plant output y_1

$\theta_1 = max\left(\theta_{11}, \theta_{11} + \theta_{22} - \theta_{21}\right)$

$z_i \rightarrow$ RHP zero of $\left(1 - G^{\circ} e^{-\Delta\theta s}\right)$

$n \rightarrow$ RHP zero number of $\left(1 - G^{\circ} e^{-\Delta\theta s}\right)$

$z_i^{\star} \rightarrow$ complex conjugate of z_i

In this manner, each of the column controller can be implemented in a proper and rational form such that plant output y_1 can be regulated independently.

The desired diagonal transfer function h_{22} is proposed in a similar manner as:

$$h_{22} = \frac{e^{-\theta_2 s}}{\alpha_2 s + 1} \prod_{i=1}^{n} \left(\frac{-s + z_i}{s + z_i^{\star}} \right) \tag{3.10}$$

where $\alpha_2 \rightarrow$ adjustable tuning parameter used for meeting system response requirement for plant output y_2

$\theta_2 = max\left(\theta_{22}, \theta_{11} + \theta_{22} - \theta_{12}\right)$

Now consider the case where $\theta_{11} + \theta_{22} \geq \theta_{12} + \theta_{21}$, then also the desired transfer functions h_{11} and h_{22} can be proposed in the similar manner as above except for the change in definitions of θ_1,θ_2 and z_i, i.e, $\theta_1 = max\left(\theta_{12}, \theta_{12} + \theta_{21} - \theta_{22}\right)$ and $\theta_2 = max\left(\theta_{21}, \theta_{12} + \theta_{21} - \theta_{11}\right)$ and z_i is the RHP zero of $1 - \dfrac{e^{-\Delta\theta s}}{G^{\circ}}$. If for both the cases, there is no RHP (Right Half Plane) zero present in the determinant of plant transfer matrix, then h_{11} and h_{22} can be proposed as:

$$h_{11} = \frac{1}{(\alpha_1 s + 1)} \cdot e^{-\theta_1 s} \tag{3.11}$$

$$h_{22} = \frac{1}{(\alpha_2 s + 1)} \cdot e^{-\theta_2 s} \tag{3.12}$$

Using inverse Laplace transform, the time domain response forms of the binary plant outputs can be given as

$$y_1(t) = \begin{cases} 0 & for\,(t \leq \theta_1) \\ 1 - e^{-\dfrac{t - \theta_1}{\alpha_1}} & for\,(t \geq \theta_1) \end{cases}$$

$$y_2(t) = \begin{cases} 0 & for\,(t \leq \theta_2) \\ 1 - e^{-\dfrac{t - \theta_2}{\alpha_2}} & for\,(t \geq \theta_2) \end{cases} \tag{3.13}$$

From above equations, it can be observed that there will be no overshoot in either of the system responses. The rise times of both the plant outputs can be derived as $t_{r1} = 2.3026\alpha_1 + \theta_1$ and $t_{r2} = 2.3026\alpha_2 + \theta_2$ respectively.

Decoupling Controller Matrix Design

The design strategy consists of two cases:

Case1: There exists no RHP zero of $det\,(G)$

Case2: There exists RHP zero of $det\,(G)$

Case 1: In this case, there is no RHP zero of determinant of G which means $1 - G^{\circ}e^{-\Delta\theta s}$ and $1 - \dfrac{e^{-\Delta\theta s}}{G^{\circ}}$ do not have any RHP zero and are stable transfer functions.

The controller matrix element c_{11} for $\theta_{11} + \theta_{22} \leq \theta_{12} + \theta_{21}$ can be derived by substituting 3.11 into 3.7.

$$c_{11} = \frac{(\tau_{11}s + 1)}{k_{11}\left(1 - G^{\circ}e^{-\Delta\theta s}\right)} \cdot \frac{e^{-(\theta_1 - \theta_{11})s}}{(\alpha_1 s + 1)}$$

The controller c_{11} can be written in a simplified form as:

$$c_{11} = \frac{(\tau_{11}s + 1)\, e^{-(\theta_1 - \theta_{11})s}}{k_{11}\,(\alpha_1 s + 1)} \cdot F \tag{3.14}$$

The practical implementation of c_{11} can be done in two parts; first part is conventional first order lead-lag controller in series with dead time compensator and second part F can be implemented using a positive feedback unit as:

Here, we can observe that the positive feedback unit ensures internal stability due to absence

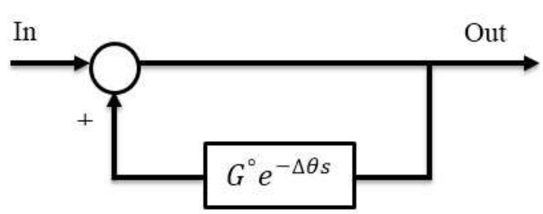


Figure 3.3: Positive feedback control unit

of RHP zeros and stability and properness of G°. In a similar way, the other controller elements can be obtained as:

$$c_{21} = -\frac{k_{21}}{k_{11}k_{22}} \cdot \frac{(\tau_{11}s + 1)\,(\tau_{22}s + 1)\, e^{-(\theta_1 + \theta_{21} - \theta_{11} - \theta_{22})s}}{(\tau_{21}s + 1)\,(\alpha_1 s + 1)} \cdot F \tag{3.15}$$

$$c_{12} = -\frac{k_{12}}{k_{11}k_{22}} \cdot \frac{(\tau_{11}s+1)(\tau_{22}s+1)\,e^{-(\theta_2+\theta_{12}-\theta_{11}-\theta_{22})s}}{(\tau_{12}s+1)(\alpha_2 s+1)} \cdot F \tag{3.16}$$

$$c_{22} = \frac{(\tau_{22}s+1)\,e^{-(\theta_2-\theta_{22})s}}{k_{22}(\alpha_2 s+1)} \cdot F \tag{3.17}$$

Now consider the case where $\theta_{11}+\theta_{22} \geq \theta_{12}+\theta_{21}$, each of the controller elements can be derived as:

$$c_{11} = -\frac{k_{22}}{k_{12}k_{21}} \cdot \frac{(\tau_{12}s+1)(\tau_{21}s+1)\,e^{-(\theta_1+\theta_{22}-\theta_{12}-\theta_{21})s}}{(\tau_{22}s+1)(\alpha_1 s+1)} \cdot F \tag{3.18}$$

$$c_{21} = \frac{(\tau_{12}s+1)\,e^{-(\theta_1-\theta_{12})s}}{k_{12}(\alpha_1 s+1)} \cdot F \tag{3.19}$$

$$c_{12} = \frac{(\tau_{21}s+1)\,e^{-(\theta_2-\theta_{21})s}}{k_{21}(\alpha_2 s+1)} \cdot F \tag{3.20}$$

$$c_{22} = -\frac{k_{11}}{k_{12}k_{21}} \cdot \frac{(\tau_{12}s+1)(\tau_{21}s+1)\,e^{-(\theta_2+\theta_{11}-\theta_{12}-\theta_{21})s}}{(\tau_{11}s+1)(\alpha_2 s+1)} \cdot F \tag{3.21}$$

where $F = \frac{1}{1-\left(\frac{e^{-\Delta\theta s}}{G^{\circ}}\right)}$ is doubly proper and stable. Therefore, it can be implemented in a similar manner as shown in fig3.3.

<u>Observations from designing of controllers:</u>

- Each column controllers of the decoupling controller matrix are exactly tuned by a single adjustable parameter.

- There exists no cross coupling in tuning each of the column controllers.

- When adjustable parameters α_1 and α_2 are tuned to zero, the system output responses will come to the ideal case, i.e., $h_1 = e^{-\theta_1 s}$ and $h_2 = e^{-\theta_2 s}$.

- This means the binary process outputs will reach the values of set-points after the process time delays $\theta_1\, and\, \theta_2$ respectively.

- Each column controller of the decoupling controller matrix will no longer be proper and cannot be physically realized.

Case 2: When RHP zeros are present in the determinant of G, then its number can be ascertained by observing the nyquist curve of $-G^{\circ}e^{-\Delta\theta s}\left(or - \frac{e^{-\Delta\theta s}}{G^{\circ}}\right)$. The number of encirclement of point $(-1,0)$ in the complex plane is equal to number of RHP zero of $det\,(G)$. Another way to determine the number of RHP zeros is by solving $1 - G^{\circ}e^{-\Delta\theta s} = 0\left(or\, 1 - \frac{e^{-\Delta\theta s}}{G^{\circ}} = 0\right)$ using MATLAB.

Consider the case $\theta_{11} + \theta_{22} \leq \theta_{12} + \theta_{21}$ for some TITO processes. Substituting 3.9 and 3.10 in 3.5, the controller elements can be derived.

$$c_{11} = \frac{(\tau_{11}s+1)\,e^{-(\theta_1-\theta_{11})s}}{k_{11}\,(\alpha_1 s+1)\prod_{i=1}^{n}(s+z_i^{\star})} \cdot D \tag{3.22}$$

$$c_{21} = -\frac{k_{21}}{k_{11}k_{22}} \cdot \frac{(\tau_{11}s+1)\,(\tau_{22}s+1)\,e^{-(\theta_1+\theta_{21}-\theta_{11}-\theta_{22})s}}{(\tau_{21}s+1)\,(\alpha_1 s+1)\prod_{i=1}^{n}(s+z_i^{\star})} \cdot D \tag{3.23}$$

$$c_{12} = -\frac{k_{12}}{k_{11}k_{22}} \cdot \frac{(\tau_{11}s+1)\,(\tau_{22}s+1)\,e^{-(\theta_2+\theta_{12}-\theta_{11}-\theta_{22})s}}{(\tau_{12}s+1)\,(\alpha_2 s+1)\prod_{i=1}^{n}(s+z_i^{\star})} \cdot D \tag{3.24}$$

$$c_{11} = \frac{(\tau_{22}s+1)\,e^{-(\theta_2-\theta_{22})s}}{k_{22}\,(\alpha_2 s+1)\prod_{i=1}^{n}(s+z_i^{\star})} \cdot D \tag{3.25}$$

where

$$D = \frac{\prod_{i=1}^{n} -s+z_i}{1-G^{\circ}e^{-\Delta\theta s}} \tag{3.26}$$

The first part of the controllers can be implemented similar to as in case1, but the second part (D) cannot be directly implemented because of pole-zero cancellation, which cannot be ignored. To overcome it, Pade expansion is used to compute linear fractional approximation of D,i.e.,

$$D_{P/Q} = \frac{\prod_{i=0^{U}a_i s^i}}{\prod_{j=0}^{V} b_j s^j} \tag{3.27}$$

where P and Q are the user-specified orders to achieve the desirable system performance specification, and the constant coefficients $a_i\,(i=1,2,...,P)$ and $b_j\,(j=1,2,...,Q)$ are determined as follows:

$$\begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_P \end{bmatrix} = \begin{bmatrix} d_0 & 0 & 0 & \cdots & 0 \\ d_1 & d_0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ d_P & d_{P-1} & d_{P-2} & \cdots & d_{P-Q} \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_Q \end{bmatrix} \tag{3.28}$$

$$\begin{bmatrix} d_P & d_{P-1} & \cdots & d_{P-Q+1} \\ d_{P+1} & d_P & \cdots & d_{P-Q+2} \\ \vdots & \vdots & \ddots & \vdots \\ d_{P+Q-1} & d_{P+Q-2} & \cdots & d_P \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_Q \end{bmatrix} = - \begin{bmatrix} d_{P+1} \\ d_{P+2} \\ \vdots \\ d_{P+Q} \end{bmatrix} \tag{3.29}$$

where $d_k(k = 0, 1, ..., P + Q)$ are the constant coefficients of each term in the Maclaurin expansion series of D_{ij}.

d_k and b_0 are given by

$$d_k = \frac{1}{k!} \lim_{s \to \infty} \frac{d^k D_{ij}}{ds^k}, k = 0, 1, ..., P + Q \tag{3.30}$$

$$b_0 = \begin{cases} +1, b_k \geq 0, \\ -1, b_k < 0 \end{cases} \tag{3.31}$$

3.28 and 3.29 can be derived by substituting 3.27 into Maclaurin series expansion of D and then compairing the constant coefficient of each complex variable with same index on both sides.

Now consider the case where $\theta_{11} + \theta_{22} \geq \theta_{12} + \theta_{21}$, following a similar design procedure as above, the controller matrix elements can be derived as:

$$c_{11} = -\frac{k_{22}}{k_{12}k_{21}} \cdot \frac{(\tau_{12}s + 1)(\tau_{21}s + 1)\, e^{-(\theta_1 + \theta_{22} - \theta_{12} - \theta_{21})s}}{(\tau_{22}s + 1)(\alpha_1 s + 1)\prod_{i=1}^{n}(s + z_i^\star)} \cdot D \tag{3.32}$$

$$c_{21} = \frac{(\tau_{12}s + 1)\, e^{-(\theta_1 - \theta_{12})s}}{k_{12}(\alpha_1 s + 1)\prod_{i=1}^{n}(s + z_i^\star)} \cdot D \tag{3.33}$$

$$c_{12} = \frac{(\tau_{21}s + 1)\, e^{-(\theta_2 - \theta_{21})s}}{k_{21}(\alpha_2 s + 1)\prod_{i=1}^{n}(s + z_i^\star)} \cdot D \tag{3.34}$$

$$c_{22} = -\frac{k_{11}}{k_{12}k_{21}} \cdot \frac{(\tau_{12}s + 1)(\tau_{21}s + 1)\, e^{-(\theta_2 + \theta_{11} - \theta_{12} - \theta_{21})s}}{(\tau_{11}s + 1)(\alpha_2 s + 1)\prod_{i=1}^{n}(s + z_i^\star)} \cdot D \tag{3.35}$$

where

$$D = \frac{\prod_{i=1}^{n} -s + z_i}{1 - \frac{e^{-\Delta\theta s}}{G^{\circ}}} \text{ (D can be approximated using Pade linear approximation formula)}$$

Observations from designing of controllers:

- 3.26 shows that $G^{\circ}e^{-\delta\theta s}$ has a tendency to decay faster than the rational numerator polynomial as $s \to \infty$. Thus, high accuracy can be attained by using a rational linear fractional approximation for D.
- In case of multiple RHP zeros we have to choose the dominant RHP zero in order to have a simpler practical decoupling controller matrix as the off-dominant RHP zeros have little impact on the achievable system performance.
- The system performance will degrade depending upon the choice of dominant RHP zero. Hence, a compromise is to be done between achievable system performance and calculation complexity of decoupling controller matrix.
- The choice of b_0 shown in 3.31 is intended to keep all of $b_j(j = 0, 1, Q)$ all of same sign, to exclude any possibility of RHP zeros from being enclosed on the denominator of 3.27.
- Higher approximation of D will improve the system performance but it can be involved with RHP poles. To check this, Routh-Hurwitz stability criterion is to be used to identify the stability of higher approximation order ($V \geq 3$) before using it in practice.

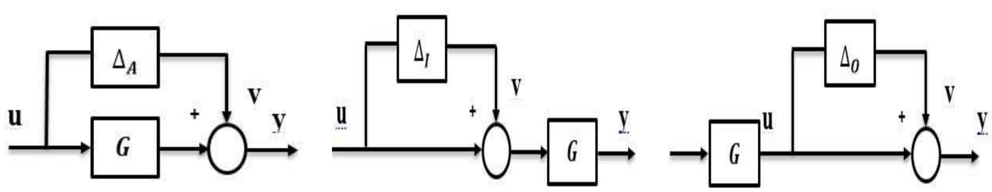
Robustness Analysis

The discussion which continued from the starting of the IMC scheme upto this point is mainly focussed on decoupling control of the plant model. This holds the stability of the nominal system output responses. But in practical there may occur uncertainties in a system which will cause deviation in the system response from its nominal value. These uncertainties are due to unmodeled dynamics of the process. Therefore, there is a need to evaluate the resultant control system robust stability so that the tuning constraints for holding the control system stability can be ascertained in presence of the process uncertainty.

The process uncertainties are mainly classified into three categories:

- Additive Uncertainty
- Multiplicative input Uncertainty
- Multiplicative output Uncertainty

The other types of process unstructured or structured uncertainties can be lumped into the above mentioned process uncertainties to cope with in practice. The perturbed control system in the form of M-Δ structure for robustness analysis is given below.



(a) additive (b) Multiplicative input (c) Multiplicative output

Figure 3.4: Process uncertainties

The equivalent relationship between the small gain theorem and the multi-variable spectral radius criterion is utilized here to formulate the robust stability constraints which can be given as:

$$\rho(C(I+GC)^{-1}\Delta_A) < 1\forall\omega\epsilon[0,\infty), \tag{3.36}$$

$$\rho(C(I+GC)^{-1}G\Delta_I) < 1\forall\omega\epsilon[0,\infty), \tag{3.37}$$

$$\rho(GC(I+GC)^{-1}\Delta_O) < 1\forall\omega\epsilon[0,\infty), \tag{3.38}$$

To ensure robust stability, the constraints defined in 3.36, 3.37 and 3.38 must be satisfied.

3.4 Internal Model Control Approach II

The IMC strategy discussed above is based on open loop transfer function of the TITO(Two-Input-Two-Output) system. The design of controller is based on the assumption that the plant and its model are same, which also accounts for the change in plant and its model by taking the difference of the plant output and model output as the feedback. This can be analysed from fig3.2. If the difference between plant and its model ($G\, and\, G_m$) in terms of plant parameters is too large, then it would be difficult to achieve satisfactory results sometimes to the extent of destabilizing the system. In this approach [11], model of the plant is not considered. The decoupling controller design solely depends upon closed loop transfer function of the MIMO (Multi Input Multi Output)system. The decoupling controller design includes these steps:

3.4.1 Decoupling Control Preconditions

- Each element of the transfer matrix should be stable. The general transfer matrix form of the MIMO process is given by [11]

$$G = \begin{bmatrix} g_{11} & \cdots & g_{1n} \\ \vdots & \vdots & \vdots \\ g_{n1} & \cdots & g_{nn} \end{bmatrix} \tag{3.39}$$

- The conventional unity feedback control structure of a MIMO system is as given below

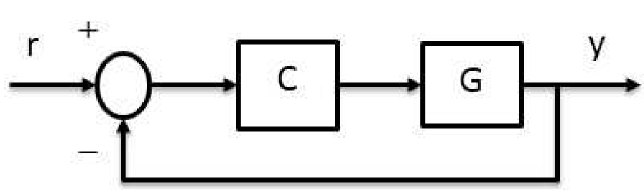


Figure 3.5: Generalized control structure

- The closed loop system transfer matrix can be determined as:

$$H = GC(I + GC)^{-1} \tag{3.40}$$

- The ideal decoupled response transfer matrix should be in the form:

$$H = \begin{bmatrix} h_{11} & 0 & \cdots & \cdots & 0 \\ 0 & h_{22} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & h_{nn} \end{bmatrix} \tag{3.41}$$

where h_{ii} is a physically proper and stable function and $det(H) \neq 0$. This requires $det(G) \neq 0$ to be satisfied.

3.4.2 Decoupling Controller Design

- The controller matrix can be obtained from 3.40 and 3.41 which can be given as:

$$C = G^{-1}(H^{-1} - I)^{-1} = \frac{adj(G)}{det(G)} diag\left[\frac{h_{ii}}{1 - h_{ii}}\right]_{nxn}$$

where $adj(G) = [G^{ij}]^T{}_{nxn}$ is the adjoint of the process transfer matrix G, and G^{ij} denotes the cofactor of each transfer element g_{ij} of G. Each element of controller matrix can be derived as:

$$c_{ji} = \frac{G^{ij}}{det(G)} . \frac{h_{ii}}{1 - h_{ii}}, i, j = 1, 2, ..., n$$

- Let us define $r_{ij} = \dfrac{G^{ij}}{det(G)} = r_{0,ij} e^{l^{ij} s}, i, j = 1, 2, ..., n$,where $r_{0,ij}$ represents the delay free part of r_{ij}.

- In order to achieve proper function, the inverse relative degree of $r_{0,ij}$ is defined as u_{ij} and is given as

$$lim_{s \to \infty} \frac{s^{u_{ij} - 1}}{r_{0,ij}} = 0$$

and let

$$U_i = max\{u_{ij}; j = 1, 2, ..., n\}, i = 1, 2, ..., n.$$

and

$$\theta_i = max\{l_{ij}; j = 1, 2, ..., n\}, i = 1, 2, ..., n.$$

- The determinant of the plant is expressed in the form

$$det(G) = \frac{\psi(s)e^{-\theta_{min}s}}{\phi(s)}$$

where $\phi(s)$ is the least common denominator of all terms of $det(G)$, and $\psi(s)$ is the corresponding numerator polynomial, in which there exists atleast one term that does not contain any time-delay and thus is rational. Apparently, $det(G)$ has same zeros with $\psi(s)$.

To achieve the H_2 optimal performance specification for system output response,the different desired forms of system response transfer matrix and decoupling controller matrix for different cases are summarized below.

Case1: $det(G)$ has no RHP zero

$$h_{ii}(i = 1, 2, ..., n) = \frac{e^{-\theta_i s}}{(\alpha_i s + 1)^{U_i}} \tag{3.42}$$

$$c_{ji}(i, j = 1, 2, ..., n) = \frac{D_{ij}e^{-(\theta_i - l_{ij}s)}}{(\alpha_i s + 1)^{U_i}} . \frac{1}{1 - \dfrac{e^{-(\theta_i s}}{(\alpha_i s + 1)^{U_i}}}, D_{ij} = r_{0,ij} \tag{3.43}$$

Case2: $det(G)$ has finite RHP zeros or infinite RHP and LHP zeros [$z_v(v = 1, 2, ..., q_i)$= the RHP zeros excluding those canceled by common RHP zeros of $G^{ij}(j = 1, 2, ..., n)$]

$$h_{ii}(i = 1, 2, ..., n) = \frac{e^{-(\theta_i s}}{(\alpha_i s + 1)^{U_i}} . \sum_{v=1}^{q_i} \frac{-s + z_v}{s + z_v^*} \tag{3.44}$$

$$
\begin{aligned}
c_{ji}(i,j=1,2,...,n) =& D_{ij}e^{-(\theta_i - l_{ij})s}(\alpha_i s+1)^{U_i}\sum_{v=1}^{q_i}\frac{-s+z_v}{s+z_v^*} \\
&.\frac{1}{1-\frac{e^{-\theta_i s}}{(\alpha_i s+1)^{U_i}}.\sum_{k=1}^{q_i}\frac{-s+z_v}{s+z_v^*}}, D_{ij} = r_{0,ij}\sum_{k=1}^{q_i}(-s+z_v)
\end{aligned} \tag{3.45}
$$

Case3: $det(G)$ has infinite RHP zeros but finite LHP zeros [$z_v(v=1,2,...,q_i)$= the RHP zeros excluding those equal to the complex conjugates of the common RHP zeros of $G^{ij}(j=1,2,...,n)$]

$$
h_{ii}(i=1,2,...,n) = \frac{e^{-(\theta_i s}}{(\alpha_i s+1)^{U_i}}.\frac{\psi(s)e^(\theta_{max}-\theta min)s}{\psi(-s)} \times \sum_{v=1}^{q_i}\frac{-s-z_v}{s-z_v^*} \tag{3.46}
$$

$$
\begin{aligned}
c_{ji}(i,j=1,2,...,n) = & \frac{G^{ij}D_{ij}\phi(s)e^{-(\theta_{min}-\theta_i)s}}{(\alpha_i s+1)^{U_i}\sum_{v=1}^{q_i}\frac{-s+z_v}{s-z_v^*}}.\frac{1}{1-\frac{D^{ij}\psi(s)e^{-\theta_i s}}{(\alpha_i s+1)^{U_i}\sum_{v=1}^{q_i}(s-z_v^*)}}, \\
D_{ij} = & \frac{e^{(\theta_{max}-\theta_{min})s}}{\psi(-s)}.\sum_{v=1}^{q_i}(s-z_v^*)
\end{aligned} \tag{3.47}
$$

It can be observed from the controller matrix forms for case 2 and 3 that there inevitably exists RHP zero-pole cancellation, which, however, cannot be directly removed from the expression. To achieve its practical implementation, a linear Pade expansion transformation is utilized to approximate D_{ij} in the controller matrix. The equations3.26 $-$ 3.31 governing the Pade expansion are defined in IMC approach I. In addition to this, the control system robust stability analysis is same as discussed in IMC approah I.

3.5 Inverted Decoupling Approach

Inverted decoupling is one of the most significant type of dynamic decoupling methods. There are some other dynamic decoupling methods like ideal decoupling, simplified decoupling. Ideal decoupling is based on inverse transfer matrix of the plant. For a proper plant transfer function inverse cannot be realized because it will be improper function. In literature, it is proved that ideal decoupling is not suitable when there is error in the model of the plant. Simplified decoupling is based on transfer function matrix but it suffers from a limitation that it needs model reduction before implementation. Inverted decoupling is the one which eliminates the limitations of both ideal and simplified decoupling.

3.5.1 General Inverted Decoupling

Inverted decoupling is based on feedforward control structure which is shown below [19]. The

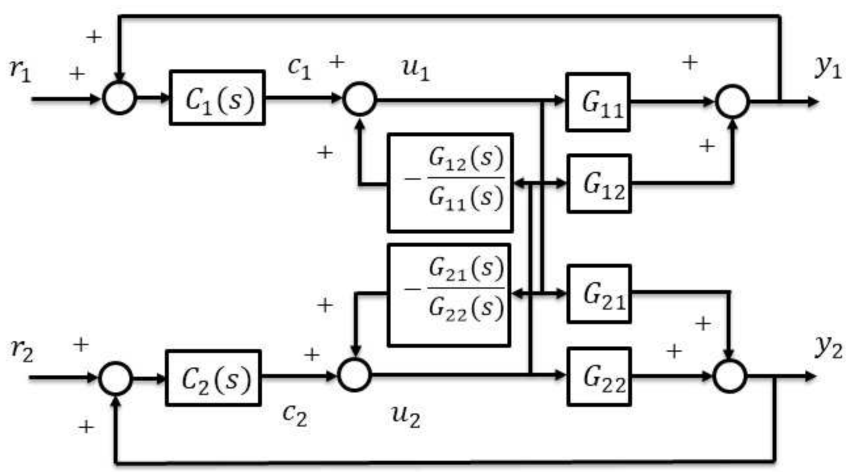


Figure 3.6: Inverted decoupling of a TITO process

above figure shows a general inverted decoupling structure for a TITO process. It consists of a controller and a decoupler. The controller design is same for SISO systems as the decoupler and plant together result in a decoupled system, thus making the controller diagonal. In fig3.6,r_1 and r_2 are the reference variables(set-points), c_1 and c_2 are the controller outputs, u_1 and u_2 are the plant inputs to the diagonal elements and y_1 and y_2 are the plant outputs respectively. The decoupler matrix can be determined from the control input equation u_1 and u_2.

$$\begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_1(s) \\ c_2(s) \end{bmatrix} + \begin{bmatrix} 0 & -G_{12}(s)/G_{11}(s) \\ -G_{21}(s)/G_{22}(s) & 0 \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} \tag{3.48}$$

$$\begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} = \begin{bmatrix} 1 & G_{12}(s)/G_{11}(s) \\ G_{21}(s)/G_{22}(s) & 1 \end{bmatrix}^{-1} \begin{bmatrix} c_1(s) \\ c_2(s) \end{bmatrix} \tag{3.49}$$

The decouler matrix can be given by:

$$D(s) = \begin{bmatrix} 1 & G_{12}(s)/G_{11}(s) \\ G_{21}(s)/G_{22}(s) & 1 \end{bmatrix}^{-1} \tag{3.50}$$

The product of plant and decoupler matrix can be given as:

$$\begin{aligned} G(s)D(s) &= \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} 1 & G_{12}(s)/G_{11}(s) \\ G_{21}(s)/G_{22}(s) & 1 \end{bmatrix}^{-1} \\ &= \begin{bmatrix} G_{11}(s) & 0 \\ 0 & G_{22}(s) \end{bmatrix} \end{aligned} \tag{3.51}$$

From 3.51, one can observe that the combination of plant and decoupler together give a diagonal transfer matrix such that SISO control techniques can be used for design of controllers. There are some important considerations involved in the design of decoupler which limits the applicability of inverted decoupling to few processes. These considerations are:

- $-G_{12}(s)/G_{11}(s)$ and $-G_{21}(s)/G_{22}(s)$ must be either strictly proper or proper functions.
- The delay of $G_{12}(s)$ must be greater than or equal to $G_{11}(s)$ and the order of $G_{12}(s)$ must be greater than or equal to $G_{11}(s)$.
- The delay of $G_{21}(s)$ must be greater than or equal to $G_{22}(s)$ and the order of $G_{21}(s)$ must be greater than or equal to $G_{22}(s)$.

If these considerations are satisfied by any plant transfer matrix. then only inverted decoupling can be employed. For time-delay compensation one can determine a dead time compensator such that the diagonal element of the plant will be having minimum time-delay in its corresponding row. This modification used with inverted decoupling redefines its naming as improved inverted decoupling where dead-time compensation matrix is used.

3.5.2 Structure of Improved Inverted Decoupling

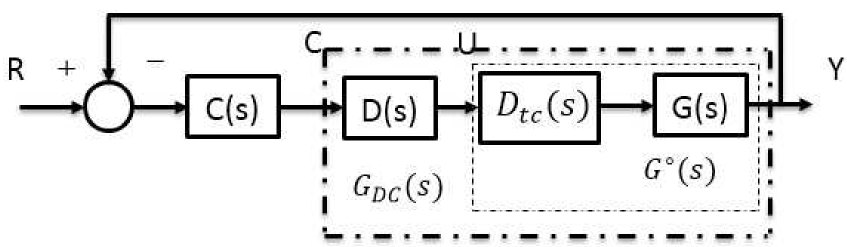


Figure 3.7: Inverted decoupling with dead time compensator

In fig3.7, $D_{tc}(s)$ represents the dead time compensation matrix which when applied to plant results in a compensated plant $G^{\circ}(s)$ in which diagonal elements are having minimum time delay. $G_{DC}(s)$ represents a dead time compensated decoupled plant. The other notations have predefined meanings. The improved inverted decoupling structure can be generalized for a MIMO process. The diagonal controller can be in PI/PID form. But here we will consider PI form of controller which is given as:

$$C(s) = diag\left[k_{P,i} + k_{I,i}/s\right], i = 1, 2, ..., n$$

$$D(s) = \left[D_{ij}(s)\right], i, j = 1, 2, ..., n$$

$$D_{tc}(s) = diag\left[e^{\tau_{ii}s}\right], i = 1, 2, ..., n$$

$$G(s) = diag\left[G_{ij}(s)\right], i, j = 1, 2, ..., n$$

3.5.3 Design of Dead time compensator

The design of D_{tc} gives a dead-time compensated plant in which $-G^{\circ}_{ij}(s)/G^{\circ}_{ii}(s)$ is physically realizable only if the plant transfer functions are defined in FOPDT(First Order Plus Dead Time) or SOPDT(Second Order Plus Dead Time) form. Another form of plant transfer function matrix is possible if order of $G_{ij}(s)$ is greater than $G_{ii}(s)$. For simplicity, we will consider TITO plant for dead-time compensator design. Consider a 2×2 system where $\Delta = [\theta_{ij}, i, j = 1, 2]$ represents the time-delay matrix of the plant, and $T_{dtc} = diag\left[\tau_{11}, \tau_{22}\right]$

represents the dead-time matrix of dead time compensator matrix($D_{tc}(s)$). The problem to find T_{dtc} is defined as:

To determine $\tau_{ii} \geq 0$, i=1,2, such that the diagonal element in the below defined matrix becomes the smallest one in its respective row.

$$\Lambda = \begin{bmatrix} \theta_{11} + \tau_{11} & \theta_{12} + \tau_{22} \\ \theta_{21} + \tau_{11} & \theta_{22} + \tau_{22} \end{bmatrix} \tag{3.52}$$

The steps involved in design of $D_{tc}(s)$ are as follows:

- Develop a search point set from time delay of the plant transfer matrix and sort them neglecting repeated terms.

$$\theta_s = \{\theta_{ij}, |\theta_{ij} - \theta_{uv}|, i, j, u, v = 1, 2\}, \theta_s = \{\theta_s, \theta_{s2}, ..., \theta_{s7}\} \tag{3.53}$$

- Ransack the set θ_s for all τ_{ii} to determine the dead-time matrix T_{dtc}.

- Check if all diagonal elements of Λ matrix are the smallest ones in their relative row.

- Determine the column of minimal value in each row of Δ matrix and define the column numbers as q_i. q_i can be a vector as both the column can have same value.

- Choose an element $n_{q_{i1}}$ or $n_{q_{i1}}, n_{q_{i2}}$ in case q_i is a vector from $q_i, i = 1, 2$ to compose a permutation $n_{q_{i1}}, n_{q_{i2}}$ such that $\prod_{i=1}^{2} n_{q_{i1}} = 2..1$.

- $D_{tc}(s) = diag\,[e^{-\tau_{ii}s}, i = 1, 2]$ and input/output pairing is $i \rightarrow q_{i1}$ or $q_{i2} (i = 1, 2)$.

- If above condition is satisfied, then there exists a permutation which gives minimal diagonal value of Λ, else the given plant cannot be compensated by dead time compensator. In that case, inverted decoupling can't be applied for such systems.

3.5.4 Decoupler and Controller design

The use of dead time compensator modifies the plant elements G_{ij} by G_{ij}°. The modified equation for decoupler can be determined as follows:

$$\begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_1(s) \\ c_2(s) \end{bmatrix} + \begin{bmatrix} 0 & -G_{12}^{\circ}(s)/G_{11}^{\circ}(s) \\ -G_{21}^{\circ}(s)/G_{22}^{\circ}(s) & 0 \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} \quad (3.54)$$

The modified decoupler matrix is given by:

$$D(s) = \begin{bmatrix} 1 & G_{12}^{\circ}(s)/G_{11}^{\circ}(s) \\ G_{21}(s)^{\circ}/G_{22}(s)^{\circ} & 1 \end{bmatrix}^{-1} \quad (3.55)$$

Now we get a decoupled plant for which SISO controllers need to be designed. There are several FOPDT/SOPDT model based PI tuning methods defined in literature. In present work, the PI control parameters are defined for a particular maximum sensitivity(M_s) for different loop as given in [1]. The plant FOPDT model and the PI controller are in the form given below.

$$G_{ij}(s) = \frac{ke^{-\theta s}}{ts+1}$$

$$C(s) = k_c(1 + \frac{1}{T_i s})$$

For above plant and controller general form, the PI parameters are given as:

$$\begin{aligned} M_s = 1.71(smooth\, control) \quad & k_c = \frac{0.40t}{K\theta} \quad T_i = t \\ M_s = 1.38(tight\, control) \quad & k_c = \frac{0.57t}{K\theta} \quad T_i = t \end{aligned} \quad (3.56)$$

3.6 Simulation Example

Consider the Wood-Berry distillation column process as defined in 2.3

$$G(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{(16.7s+1)} & \dfrac{-18.9e^{-3s}}{(21s+1)} \\ \dfrac{6.6e^{-7s}}{(21s+1)} & \dfrac{-19.4e^{-3s}}{(14.4s+1)} \end{bmatrix}$$

3.6.1 Design steps for IMC approach 1

Step1: Comparing with standard form of transfer function for TITO system as given in 3.1, we get

$\theta_{11} + \theta_{22} = 4$, $\theta_{12} + \theta_{21} = 10$ Here, $\theta_{11} + \theta_{22} \leq \theta_{12} + \theta_{21}$.

Step2: Determine determinant of G and draw Nyquist plot of $-G^{\circ}e^{-\Delta\theta s}$ to check the presence of RHP zero. Step3: From Nyquist plot, it can be observed that there is no RHP zero located

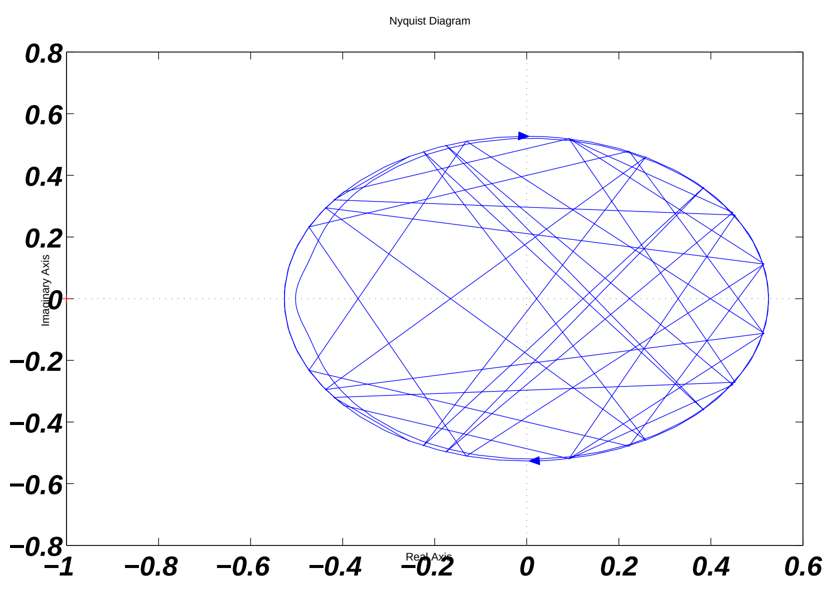


Figure 3.8: Nyquist plot of $-G^{\circ}e^{-\Delta\theta s}$

in $-G^{\circ}e^{-\Delta\theta s}$. Therefore, the design of controller will be following case1 procedure.

Step4: The controller matrix can be obtained from equations $(3.14 - 3.17)$ as:

$$C = F \begin{bmatrix} \dfrac{(16.7s+1)}{12.8(\alpha_1 s+1)} & \dfrac{-0.0761(16.7s+1)(14.4s+1)e^{-2s}}{(21s+1)(\alpha_2 s+1)} \\ \dfrac{0.0266(16.7s+1)(14.4s+1)e^{-4s}}{(10.9s+1)(\alpha_1 s+1)} & \dfrac{-(14.4s+1)}{19.4(\alpha_2 s+1)} \end{bmatrix}$$

where

$$F = \frac{1}{1 - \dfrac{0.5023(16.7s+1)(14.4s+1)}{(21s+1)(10.9s+1)} e^{-6s}}$$

F can implemented using fig3.3.

Step5: By using Pade approximation formulas $3.28 - 3.30$, the second order approximation of F obtained is given as:

$$F_{2/2} = \frac{73.648s^2 + 51.077s + 2.01}{150.662s^2 + 320.283s + 1}$$

Step6: Substitute the second order approximation of F in the controller to obtain the simulation results.

3.6.2 Design steps for IMC approach 2

Step1: Determine the number of RHP zeros of determinant of $G(s)$ by observing nyquist plot. From 3.8, it is observed that there are no RHP zeros present in the determinant of $G(s)$.

Step2: As there are no RHP zeros, the desired closed loop transfer function and the controller design formulas will be as given in case13.42 and 3.43.

Step3: Determine r_{ij},$r_{0,ij}$,U_i and θ_i.

Step4: Use Pade approximation$(3.28 - 3.30)$ for $r_{0,ij}$. Choose P and Q such that the controller matrix elements should be in all pass form. Here, we have chosen P=2, Q=1.

Step5: The controller matrix obtained is given by:

$$C = \begin{bmatrix} \dfrac{(48.8327s^2 + 5.2622s + 0.157)}{(17.6517s+1)(\alpha_1 s+1)} F_1 & -\dfrac{(40.7154s^2 + 4.8419s + 0.1529)}{(22.3722s+1)(\alpha_2 s+1)} F_2 \\ \dfrac{(15.5065s^2 + 1.7193s + 0.0534)e^{-4s}}{(12.8017s+1)(\alpha_1 s+1)} F_1 & -\dfrac{(30.5231s^2 + 3.4512s + 0.1036)}{(19.7234s+1)(\alpha_1 s+1)} F_2 \end{bmatrix}$$

3.6.3 Design steps for Inverted decoupling

Step1: Determine dead time compensator matrix $D_{tc}(s)$ by following steps involved in design of $D_{tc}(s)$. Step2: Obtain decoupler matrix by 3.55 by using compensated plant matrix $G^{\circ}ij$. Step3: The decoupler obtained is given by:

$$D(s) = \begin{bmatrix} 1 & -\dfrac{18.9(16.7s+1)}{12.8(21s+1)} \\ -\dfrac{6.6(14.4s+1)}{19.4(10.9s+1)} & 1 \end{bmatrix}^{-1}$$

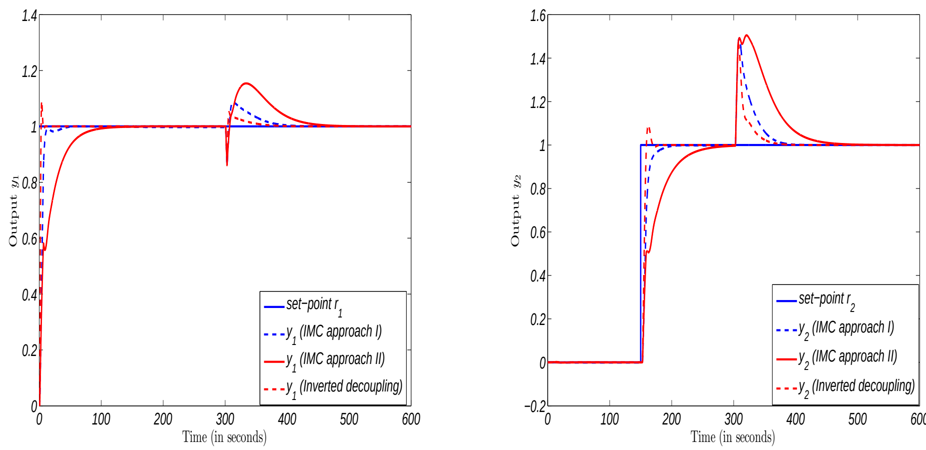
Step4: Obtain the PI controller parameters from [1]. The PI controller parameters obtained are:

$$K_{P1} = 0.7438, K_{I1} = 0.0445$$

$$K_{P2} = -0.141, K_{I2} = -0.0098$$

3.7 Results and Discussion

The simulation results of the three approaches applied to wood berry distillation column process discussed so far are given below. A unit step input is applied at t=0 second and t=150 seconds to the reference inputs respectively. The tuning parameter chosen for IMC approach I is $\alpha_1 = 4$ and $\alpha_2 = 6$ respectively. The tuning parameter for IMC approach II is $\alpha_1 = 15$ and $\alpha_2 = 20$ respectively. The simulation is configured to run at fixed step size of 0.02 in ode5 solver throughout the simulation results presented in the thesis. The following observations can be made from the simulation results.



(a) Output y_1 (b) Output y_2

Figure 3.9: Nominal system responses

- The step response of each of the dynamic decoupling methods is able to track the set-point effectively.
- The IMC approach I and inverted decoupling approach seem to be have less rise time as compared to the IMC approach II.
- IMC approach II is tuned for higher value of tuning parameter as compared to IMC approach I. Due to this, the time constant of IMC approach becomes more than that of IMC approach II. Another reason is that IMC approach 2 is close loop approach unlike IMC approach I.
- The choice of tuning parameter is based on the analysis done from set-point responses for different sets of tuning parameters. This is basically a trial and error method.
- A small value of tuning parameter(α_1 and α_2) accounts for lesser rise time or faster response at the cost of higher output energy of decoupling controller. In practical scenario, this is not accepted as the large variations in the controller outputs may wear out the actuators. The set-point response becomes oscillatory for lesser value of tuning parameter.

- A large value of tuning parameter results in slower response and smaller output energy of decoupling controller. The oscillations decrease as we increase the tuning parameter. The choice of tuning parameter is a sacrifice between system performance and output capacities of the controller.

- In both the IMC approaches it is observed that there is no overshoot in the plant outputs. This is because of the definition of desired closed loop transfer function h_{ii}. 3.3 defines the time domain response which illustrates the absence of overshoot in plant outputs.

- However, there is a small overshoot of about 10 percent in inverted decoupling approach due to the design parameters of PI controller. The change in integral gain vary the overshoot present in the set-point response.

- To check the restoring capacity of these approaches, a disturbance(inverse step) of magnitude 0.1 is applied at t=300 seconds to both the plant inputs.

- The inverted decoupling approach gives better disturbance rejection as compared to the IMC approaches but at the cost of oscillations in control output. The change in magnitude of the response is small due to which less time is required to regain its path.

- In comparison between the two IMC approaches, it is observed that IMC approach I more efficiently rejects the disturbance. This is because only feedback in the system is the disturbance. In IMC approach II, the output along with disturbance is fed back together.

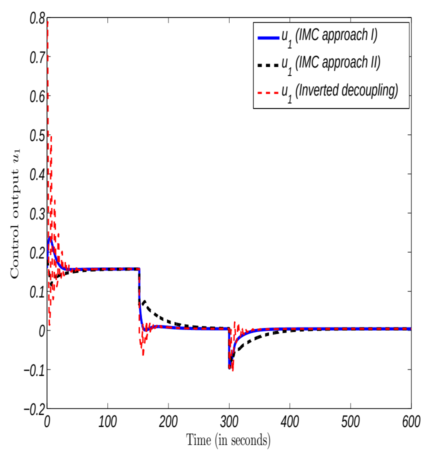
- Another important observation drawn is that when disturbance is applied, the output y_1 first decreases and then increases to settle down to its nominal value. While the output y_2 increases and then settles down to its nominal value.

- This peculiar behaviour is due to the plant model transfer function elements3.39. The control output at the instant of disturbance implication is negative ($u_1 = -0.095$ and $u_2 = -0.17$). The time delay of $g_{12} = \dfrac{-18.9e^{-3s}}{(21s+1)}$ is 3 seconds and that of $g_{11} = \dfrac{12.8e^{-s}}{(167s+1)}$ is 1 seconds and u_1 and u_2 are negative. The magnitude of g_{12} is more than that of g_{11}.

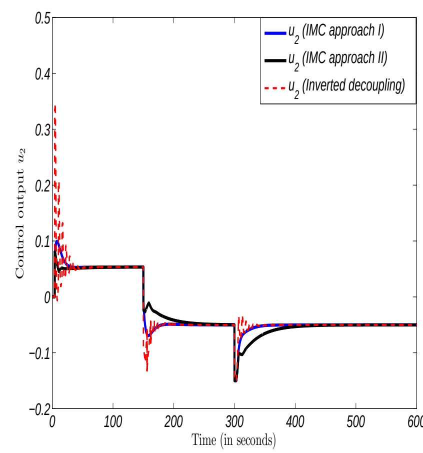
- As we know $y_1 = g_{11}u_1 + g_{12}u_2$, one can easily observe that second term of y_1 is positive and first term is negative. So, at t=301 second, only first term is active and is negative also, thus making y_1 to go below the nominal value. At t=303 seconds, the second term also becomes active and is positive enough to raise the output y_1 above the nominal value.

- A similar analysis can be done for output $y_2 = g_{21}u_1 + g_{22}u_2$. Here also first term is positive and second term is negative. As $g_{21} = \dfrac{6.6e^{-7s}}{(10.9s+1)}$ and $g_{22} = \dfrac{-19.4e^{-3s}}{(14.4s+1)}$, the magnitude of g_{22} is much more than that of g_{21}.

- At t=303 seconds, the second term of y_2 becomes active and is responsible for raising the output y_2 above the nominal value. At t=307 seconds, the first term which is negative becomes active and accounts for the decrease in output y_2 to some extent as its magnitude is much less than first term.



(a) Control output u_1

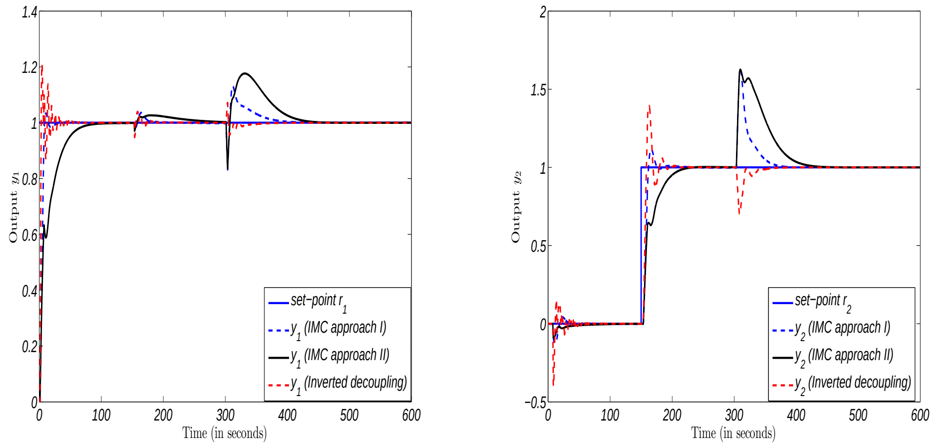


(b) Control output u_2

Figure 3.10: Nominal system control outputs

- From fig3.10, it is observed that the control outputs in inverted decoupling approach is more oscillatory as compared to both the IMC approach.

- Finally a very desired feature of control system that is robust stability of each approach is checked by introducing additive uncertainty. The plant parameters are uncertain and varied about 20-30 percent of their nominal values.



(a) Perturbed output y_1 (b) Perturbed output y_2

Figure 3.11: Perturbed system responses

- From fig3.11, it is observed that both the IMC approaches decouple the system much more robustness. The inverted decoupling approach is oscillatory at instant reference input is applied.

- From above analysis, it can be concluded that IMC approaches outperform in terms of effective decoupling, less oscillatory control outputs, smooth disturbance rejection and better robust stability, whereas inverted decoupling gives faster response and load disturbance rejection but at the cost of frequent oscillations which increase with the uncertainty in plant models. IMC approach II will be preferred over IMC approach I because IMC approach I is an open loop approach which limits the performance when there is much difference between plant and its model. In that case it may or may not be able to decouple or even control the system.

Chapter 4

Decoupling Control of Coupled Tank System

4.1 Introduction

Since last two decades, the control of coupled tank liquid level system has attracted attention of many researchers around the world. It is one of the most challenging benchmark control problems due to its non−linear and non−minimum phase characteristics. The control objective in a coupled tank system is that a desired liquid level of the liquid in tank is to be maintained when there is an inflow and outflow of water out of the tank respectively. The coupled tank system is a multi-input multi output system (MIMO) with control voltage as input and water level as the output.

Figure4.1 depicts a basic representation of a typical liquid level system. Even though the coupled tank system is simple from construction point of view but there lies a lot of control challenge owing to following characteristics:

- Non−linearity
- Non−minimum phase zeros
- Large time-delay

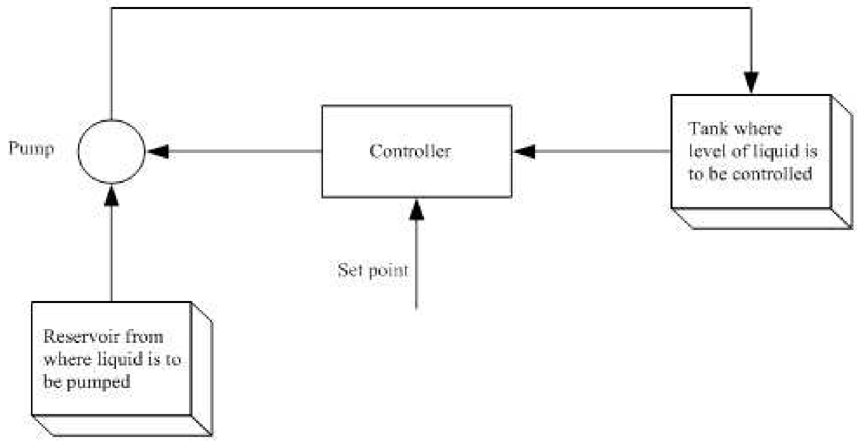


Figure 4.1: Block diagram of a typical liquid level system

There are several industrial applications of coupled tank system. Some of them are illustrated in fig4.2.

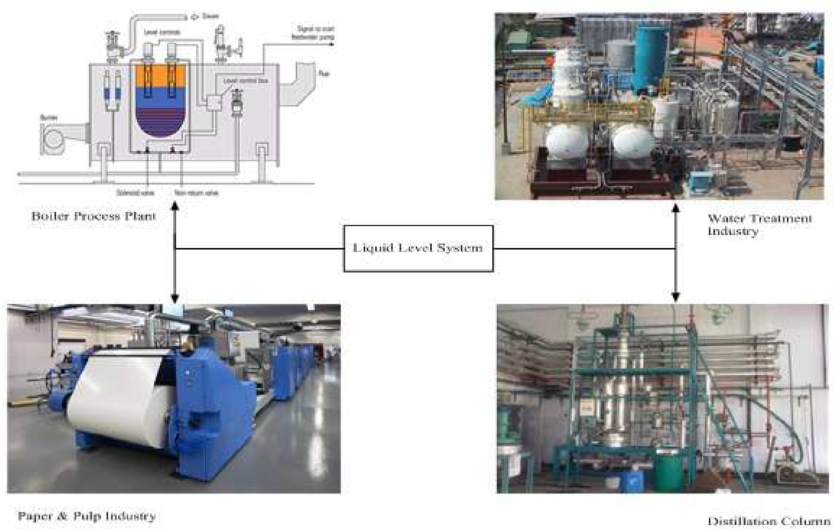


Figure 4.2: Applications of coupled tank system

4.1.1 Description of Coupled Tank System

Fig4.3 illustrates the basic schematic representation of a coupled tank system. It consists of four translucent tanks and each tank is fitted with an outlet pipe in order to transmit the over

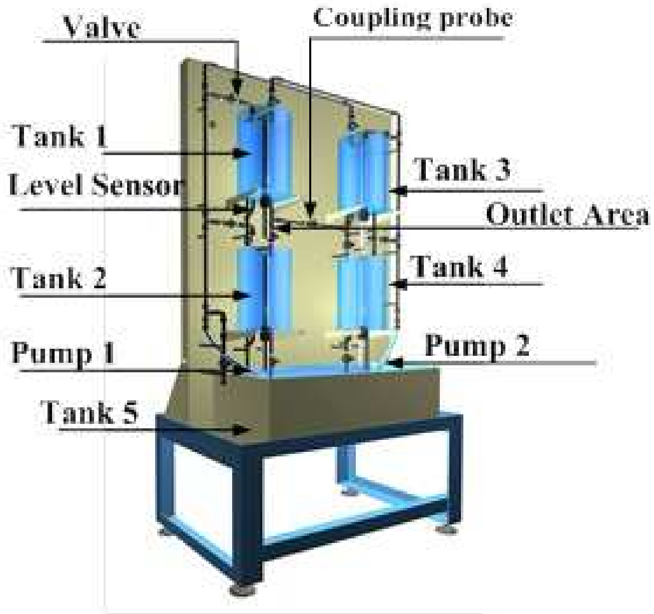


Figure 4.3: Schematic of coupled tank mechanical unit [5]

flow water to reservoir. In this process, the bottom tank ($tank5$) is used for water storage purposes i.e. as a reservoir. A level sensor is also attached at the base of each tank in order to measure the water level of the corresponding tank. The output of the level sensor is converted to $0-5$ volt DC with the help of a signal conditioning circuit. There are two pumps installed in the reservoir in order to drive the water from the bottom to the top of the tank. A scale is attached in front of all the individual tanks for the purpose of monitoring the water level. It works under two basic modes of operations i.e. local mode and remote mode. In local mode, two tanks are controlled by two separate potentiometers which are applied to two tanks to drive water to respective tanks. There is a coupling probe between the tanks which represents the interaction present in this system. The liquid level in the tanks change due to flow of water from a tank at high level to a tank at low level. The present work is focussed on local mode operation of coupled tank such that it represents a TITO(Two Input Two Output) system.

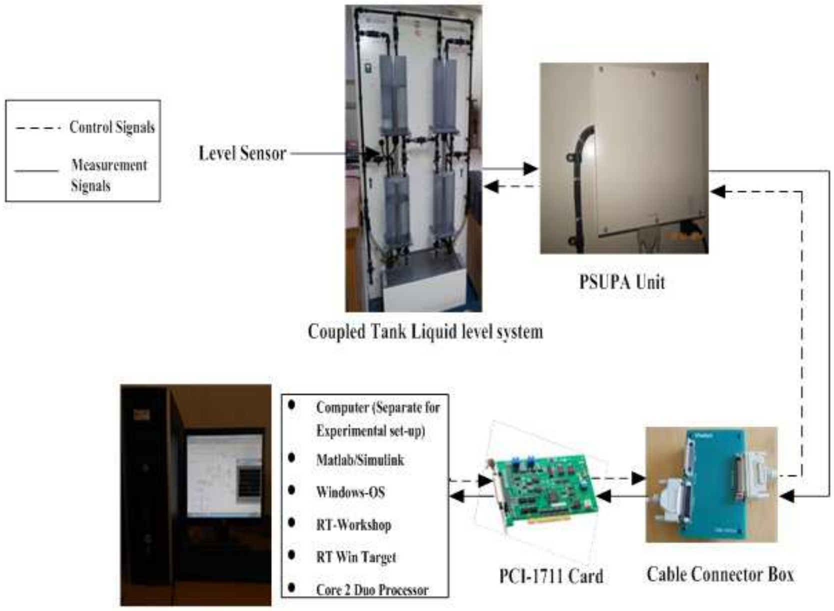


Figure 4.4: Experimental setup of coupled tank system [5]

4.1.2 Description of Experimental Setup of Coupled Tank System

Apart from the mechanical parts of the coupled tank system it is also equipped with a power supply unit and a power amplifier (PSUPA) and a cable connector box which is shown in 4.4. In this setup, PC with Advantech card and MATLAB/SIMULINK environment serve as the main control unit. Basically, the PSUPA unit amplifies the water pressure level signals and passes them as analogue signals to the PCI1711 DAQ card. Control signals to the pumps can be sent from the PC through the DAQ (PCI1711) card and PSUPA unit. The control signals, which are between 0-5 volt, are transferred to the PSUPA unit where they are transformed into 24V PWM signals in order to drive the pumps.

4.2 Problem Definition

Consider a general representation of coupled tank system operating in local mode. The model transfer function of the coupled tank system is determined by system identification. The model

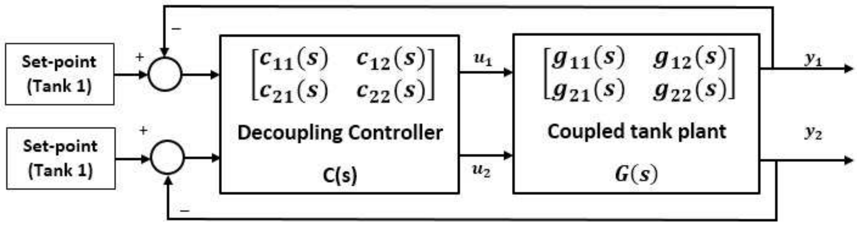


Figure 4.5: General coupled tank control system structure

transfer function is given as:

$$G(s) = \begin{bmatrix} \dfrac{2.1947e^{-5.5s}}{(615s+1)} & \dfrac{2.3e^{-9.5349s}}{614.64s+1} \\ \dfrac{2.62e^{-30s}}{(601.84s+1)} & \dfrac{2.82e^{-30s}}{(602s+1)} \end{bmatrix} \tag{4.1}$$

The control problem here is to design a decoupling controller to attain desired level of liquid in tank 1 and tank 3, when there is inflow and outflow of water from the tank irrespective of the effect of coupling probe. This means that the liquid level in tank 1 and tank 3 must be independent of the change in level of either tank desired level in the presence of coupling probe.

4.3 Decoupler Design by IMC Approach

For the model transfer function shown in 4.1, the decoupler design by IMC approach includes these steps.

- Check if the decoupling control preconditions satisfy for the plant model. From 4.1, it is observed that all the transfer function elements are stable and determinant of $G(s) \neq 0$.

- Determine the determinant of the plant model. For simplicity, we remove the s variable.

$$det(G) = \frac{\left[6.1891(614.64s+1)(601.84s+1) - 6.026(615s+1)(602s+1)e^{-4.0349s}\right]}{(615s+1)(602s+1)(614.64s+1)(601.84s+1)}$$

- Check the number of RHP zeros of determinant of $G(s)$ by Nyquist plot and location of RHP zeros by pole−zero plot.

- From Nyquist and pole−zero plot, it is observed that there is RHP zero present at $s = 0.0845 + j0.0488$ and $s = 0.0845 - j0.0488$. Since finite number of RHP zeros, we apply case2 [11] for controller design.

- Determine $r_{ij} = \dfrac{G^{ij}}{det(G)} = r_{0,ij}e^{l^{ij}s}, i, j = 1, 2$ from the determinant defined above.

- Determine u_{ij} (inverse relative degree of $r_{0,ij}$), U_i and θ_i.

$$lim_{s\to\infty}\frac{s^{u_{ij}-1}}{r_{0,ij}} = 0$$

$$U_i = max\{u_{ij}; j = 1, 2, ..., n\}, i = 1, 2, ..., n.$$

$$\theta_i = max\{l_{ij}; j = 1, 2, ..., n\}, i = 1, 2, ..., n.$$

By using above equations, we get $U_1 = 1$, $U_2 = 1$, $\theta_1 = 5.5$ and $\theta_2 = 30$.

- As there are finite RHP, the controller matrix can be derived as per case2 [11].

$$c_{ji}(i, j = 1, 2, ..., n) = D_{ij}e^{-(\theta_i - l_{ij})s}(\alpha_i s + 1)^{U_i}\sum_{v=1}^{q_i}\frac{-s + z_v}{s + z_v^*}.\frac{1}{1 - \dfrac{e^{-\theta_i s}}{(\alpha_i s + 1)^{U_i}}.\displaystyle\sum_{k=1}^{q_i}\frac{-s + z_v}{s + z_v^*}},$$

$$D_{ij} = r_{0,ij}\textstyle\sum_{k=1}^{q_i}(-s + z_v)$$

- To achieve a practical implementation of controller, a first order Pade approximation of D_{ij} is done.

$$D_{11} = \frac{(104.325s + 0.1646)}{(130.8322s + 1)}$$

$$D_{12} = -\frac{(96.9587s + 0.153)}{(130.822s + 1)}$$

$$D_{21} = -\frac{(83.4634s + 0.1343)}{(131.324s + 1)}$$

$$D_{22} = \frac{(79.5899s + 0.1281)}{(131.3289s + 1)}$$

- The controller matrix elements after Pade approximation are as follows:

$$C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

$$c_{11} = \frac{(104.325s + 0.1646)}{(130.8322s + 1)(\alpha_1 s + 1)(s^2 + 0.169s + 0.0095)}.F_1$$

$$c_{12} = -\frac{(83.4634s + 0.1343)e^{-4.0349s}}{(131.324s + 1)(\alpha_2 s + 1)(s^2 + 0.169s + 0.0095)}.F_2$$

$$c_{21} = -\frac{(96.9587s + 0.153)}{(130.822s + 1)(\alpha_1 s + 1)(s^2 + 0.169s + 0.0095)}.F_1$$

$$c_{22} = \frac{(79.5899s + 0.1281)}{(131.3289s + 1)(\alpha_2 s + 1)(s^2 + 0.169s + 0.0095)}.F_2$$

where

$$F_1 = \frac{1}{1 - \frac{e^{-5.5s}(s^2 - 0.169s + 0.0095)}{(\alpha_1 s + 1)(s^2 + 0.169s + 0.0095)}}$$

$$F_2 = \frac{1}{1 - \frac{e^{-30s}(s^2 - 0.169s + 0.0095)}{(\alpha_2 s + 1)(s^2 + 0.169s + 0.0095)}}$$

- Adjust the tuning parameters α_1 and α_2 by taking different set of observations. The tuning parameters chosen in this design are $\alpha_1 = 30$ and $\alpha_2 = 40$.

4.4 Decoupler Design by Inverted Decoupling Approach

For the model transfer function shown in 4.1, the decoupler design by Inverted decoupling approach include these steps:

- First check if the time-delay of diagonal element in plant model is minimum in its corresponding row. In our system model, the diagonal elements are possessing minimum time-delay. Therefore, there is no need to design dead time compensator in this case.

- The decoupler matrix is obtained from the equation below.

$$D(s) = \begin{bmatrix} 1 & G_{12}(s)/G_{11}(s) \\ G_{21}(s)/G_{22}(s) & 1 \end{bmatrix}^{-1}$$

- The decoupler matrix is given by:

$$D(s) = \begin{bmatrix} 1 & \frac{2.3(615s+1)e{-}4.0349s}{2.1947(614.64s+1)} \\ \frac{2.62(602s+1)}{2.82(601.84s+1)} & 1 \end{bmatrix}^{-1}$$

- After employing decoupler, a PI controller is designed for the decoupled plant which is given by:

$$Q(s) = \begin{bmatrix} \frac{2.1947e^{-5.5s}}{(615s+1)} & 0 \\ 0 & \frac{2.82e^{-30s}}{(602s+1)} \end{bmatrix}$$

- For a plant model of form $\frac{Ke^{-\theta s}}{1+st}$, the controller is given by

$$C(s) = K_c(1 + \frac{1}{T_i})$$

- The PI controller design formulas are as given in [1].

$$k_c = \frac{0.40t}{K\theta}$$

$$T_i = t$$

- The PI controller parameters obtained are as follows:

$$K_{c1} = 20.9737, T_{i1} = 615$$

$$K_{c2} = 2.8463, T_{i2} = 602$$

- The PI controller parameters can also be designed using some other FOPDT model SISO design method.

4.5 Simulation Results and Discussion

The decoupling controller designed for the coupled tank system using IMC approach and inverted decoupling approach is first simulated in MATLAB software to compare their performances. The tuning parameter values in IMC approach are $\alpha_1 = 30$ and $\alpha_2 = 60$. The simulation result is shown below.

The following observation can be drawn from fig4.6.

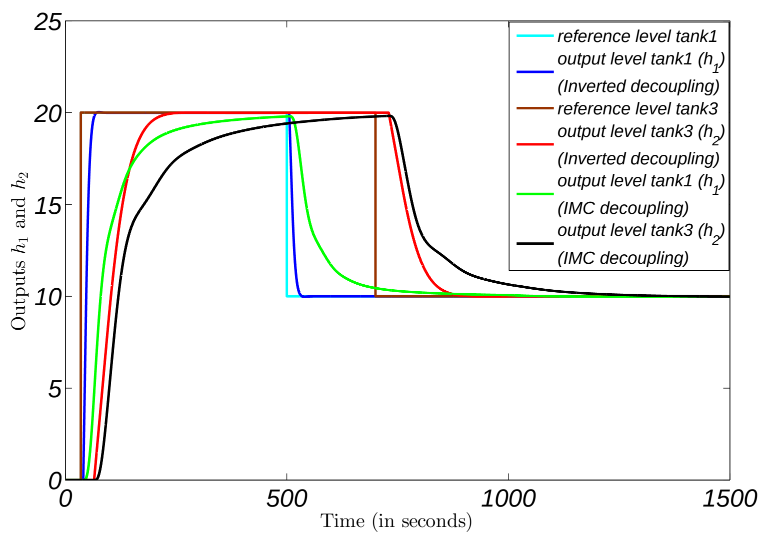


Figure 4.6: Level of tank 1 h_1 and tank 3 h_2 of coupled tank system

- A step input of amplitude 20 is applied as set-point to both the tanks of the coupled tank system at t=35 seconds. The set-point of 20 actually refers to the desired level of tank set at 20cm. It is observed that inverted decoupling approach gives faster set-point tracking as compared to IMC approach.

- To ensure the decoupling capability of both the approaches, a change in desired level of tank is introduced.

- The desired level of tank 1 is changed from 20cm to 10cm at t=500 seconds. The inverted decoupling approach provides faster reference tracking as compared to the IMC approach.
- In a similar way, the desired level of tank 3 is changed to 10cm at t=700 seconds. In this case also, the inverted decoupling provides faster reference tracking as compared to the IMC approach.
- The main objective of the decoupling controller in a multivariable system is to decouple with utmost efficiency. In context of decoupling, both the approaches are able to decouple the coupled tank system.
- The slower response of IMC approach is due to the choice of tuning parameter involved in its design. A compromise is to made between decoupling and performance of the system.
- A very need of any good controller is to provide desired performance within acceptable control output limits. This feature is much important as the large variations in control outputs will lead to damage the actuators.
- The inverted decoupling approach which seems to provide better decoupling and performance as compared to IMC approach actually requires a large control output as compared to IMC approach.

4.6 Experimental Results and Discussion

The decoupler design of coupled tank system is discussed till now in theory and simulated in MATLAB. But a true check of the controller performance can only be analysed by its application to real time system. To achieve this, we have performed decoupling control of liquid level of coupled tank system in real time. The experimental results are shown below.

The following observations can drawn from fig4.7.

- The experimental result is shown from t=200 seconds because initially a control output of 5 volts is supplied to both the tank inputs in order to fill the tank to a certain level. If anyhow we apply a desired set-point to either of the tank before 35 seconds, the pump will start at t=0 seconds. This is due to internal adjustment of the experimental setup.

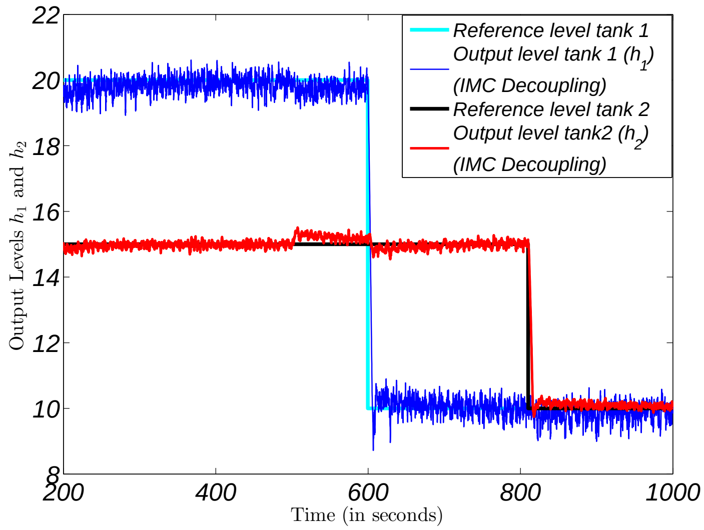


Figure 4.7: Level of tank 1 h_1 and tank 3 h_2 : IMC approach

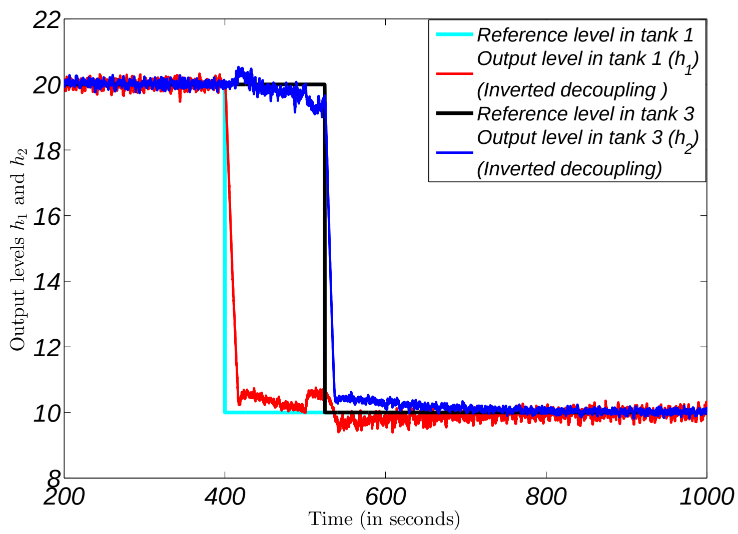


Figure 4.8: Level of tank 1 h_1 and tank 3 h_2 : inverted decoupling approach

- At t=600 seconds, the desired level of tank 1 is shifted to 15cm and it is found that the output level h_1 in IMC approach is able to track the change in its desired level more efficiently than in inverted decoupling approach without affecting output level h_2.

- At t=800 seconds, the desired level of tank 3 is changed to 15cm for IMC approach and to $10cm$ in inverted decoupling approach respectively. It is found that the output level $h_2$in IMC approach is able to track the change in its desired level more efficiently than in without affecting output level h_1.

- As observed from fig4.7 and fig4.8, the IMC approach provides better decoupling regulation as compared to inverted decoupling approach. In inverted decoupling, there is a small drift in the output level h_1 at the time of change in desired level of tank 3.

- There is chattering phenomena observed in both the approaches. This is due to application of limiter in the real time system to limit the control output within the range of 5 volts in order to prevent any damage to actuators. A limiter is necessary because a large variation in control output exists for about 600 seconds and then settle down to a value beyond the acceptable range of control output.This variation is more in inverted decoupling approach as compared to IMC approach. This may be due to the uncertainty in plant model. As we have earlier shown in previous chapter that inverted decoupling is much sensitive to uncertainties and this results in oscillatory behaviour.

- If we compare the simulation and experimental results, there is much difference in terms of set-point tracking and decoupling regulation of both the approaches. The IMC approach performs better in experimental results because of low values of control outputs as compared to inverted decoupling approach. The application of limiter introduces less variation from actual control outputs(as in IMC approach), if the actual control outputs are near to the acceptable range in real time. Thus, in real time IMC approach outperform as compared to inverted decoupling approach in all respects.

Chapter 5

Conclusion and Future Scope

5.1 Discussion and Conclusion

Decoupling of any industrial process demands not only removing the coupling effect present, but also to achieve the desired performance when there exists load disturbances. In presence of uncertainties in the model parameters, the decoupler designed should be capable to ensure robust stability. From the approaches discussed in the thesis so far, the inverted decoupling approach outperforms in comparison to others on the basis of simulation results, but it is limited to processes which possess these features: diagonal element should have minimum time delay or dead time compensation matrix can be designed for such systems and the order of diagonal element should be less than that of non-diagonal elements to ensure practical implementation of this approach. In inverted decoupling approach, the control outputs are of higher value and exhibit large variations which is unacceptable in most of the industrial processes. From experimental results, it is observed that IMC decoupling scheme is more superior to decoupling schemes. This is justified from the experimental results of coupled tank system. However, it suffers from a limitation that there is a compromise between decoupling regulation and performance (i.e. rise time, settling time) in selecting the tuning parameter. If the tuning parameter is chosen to be of low value then the response of the system will be fast but the control outputs becomes much more aggressive in nature and vice-versa. Finally, it can be concluded that IMC decoupling(closed loop approach) is the most suitable decoupling controller design approach in all practical aspects.

5.2 Future Scope

The present work puts much scope in designing a controller with lowest possible control outputs which can perform both decoupling and controlling functions with utmost robust stability and auto−tuning algorithm to adjust the tuning parameters online in real time. The present work can extended to:

- H_2 decoupling controller design for a multivariable system. The control outputs can be lowered if the H_2 norm of the system is minimized.
- State space approach to decouple time delay systems. The interactions are a result of the states which affects more than one output. If we can separate the states affecting a particular output, then it will lead to decouple the system.

Bibliography

[1] A. Ali and S. Majhi, "Pi/pid controller design based on imc and percentage overshoot specification to controller setpoint change," *ISA transactions*, vol. 48, no. 1, pp. 10–15, 2009.

[2] K. Astrom, K. Johansson, and Q.-G. Wang, "Design of decoupled pi controllers for two-by-two systems," *Control Theory and Applications, IEE Proceedings -*, vol. 149, no. 1, pp. 74–81, Jan 2002.

[3] J. Descusse, "Block noninteracting control with (non)regular static state feedback: A complete solution," *Automatica*, vol. 27, no. 5, pp. 883–886, 1991.

[4] P. L. Falb and W. Wolovich, "Decoupling in the design and synthesis of multivariable control systems," *Automatic Control, IEEE Transactions on*, vol. 12, no. 6, pp. 651–659, 1967.

[5] *Coupled tank system control experiment manual*, Feedback Instruments Ltd, 2012.

[6] J. Lee, D. Hyun Kim, and T. F. Edgar, "Static decouplers for control of multivariable processes," *AIChE Journal*, vol. 51, no. 10, pp. 2712–2720, 2005.

[7] C.-A. Lin and T.-F. Hsieh, "Stabilization, parametrization, and decoupling controller design for linear multivariable systems," in *Decision and Control, 1991., Proceedings of the 30th IEEE Conference on*, Dec 1991, pp. 563–568 vol.1.

[8] A. Linnemann and Q.-G. Wang, "Block decoupling with stability by unity output feedbacksolution and performance limitations," *Automatica*, vol. 29, no. 3, pp. 735–744, 1993.

[9] G. Liu, Z. Wang, C. Mei, and Y. Ding, "A review of decouling control based on multiple models," *Proceedings of 24th Chinese Control and Decision Conference*, 2013.

[10] T. Liu, W. Zhang, and D. Gu, "Analytical design of decoupling internal model control (imc) scheme for two-inputtwo-output (tito) processes with time delays," *Industrial Engineering Chemistry Research*, vol. 45, no. 9, pp. 3149–3160, 2006.

[11] ——, "Analytical decoupling control strategy using a unity feedback control structure for mimo processes with time delays," *Industrial Engineering Chemistry Research*, vol. 17, pp. 173–186, 2007.

[12] A. Maxim, C.-M. Ionescu, C. Copot, R. De Keyser, and C. Lazar, "Multivariable model-based control strategies for level control in a quadruple tank process," in *System Theory, Control and Computing (ICSTCC), 2013 17th International Conference*. IEEE, 2013, pp. 343–348.

[13] J. Morgan, B., "The synthesis of linear multivariable systems by state-variable feedback," *Automatic Control, IEEE Transactions on*, vol. 9, no. 4, pp. 405–411, Oct 1964.

[14] C. Rajapandiyan and M. Chidambaram, "Controller design for mimo processes based on simple decoupled equivalent transfer functions and simplified decoupler," *Industrial & Engineering Chemistry Research*, vol. 51, no. 38, pp. 12 398–12 410, 2012.

[15] M. Tham, "Multivariable control: An introduction to decoupling control," *Department of Chemical and Process Engineering, University of Newcastle Tyne*, 1999.

[16] H. Tsien, *Engineering Cybernetics*. McGraw-Hill, 1954.

[17] Q.-G. Wang, "Decoupling with internal stability for unity output feedback systems," *Automatica*, vol. 28, no. 2, pp. 411–415, 1992.

[18] Q.-G. Wang, B. Huang, and X. Guo, "Auto-tuning of tito decoupling controllers from step tests," *ISA transactions*, vol. 39, no. 4, pp. 407–418, 2000.

[19] L. Yunhui, L. Hongbo, and J. Lei, "Improved inverted decoupling control using dead-time compensator for mimo processes," in *Control Conference (CCC), 2010 29th Chinese*. IEEE, 2010, pp. 3548–3553.